Probabilistic Argumentation with Incomplete Information

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Abstract. We consider augmenting abstract argumentation frameworks with probabilistic information and discuss different constraints to obtain meaningful probabilistic information. Moreover, we investigate the problem of incomplete probability assignments and propose a solution for completing these assignments by applying the principle of *maximum entropy*.

1 Introduction

Abstract argumentation, as proposed by Dung [1], provides a simple and appealing representation in the form of a directed graph. An *abstract argumentation framework* AF is a tuple $\mathsf{AF} = (\mathsf{Arg}, \rightarrow)$ where Arg is a set of arguments and $\rightarrow$ is a relation $\rightarrow \subseteq \mathsf{Arg} \times \mathsf{Arg}$. For two arguments $\mathcal{A}, \mathcal{B} \in \mathsf{Arg}$ the relation $\mathcal{A} \rightarrow \mathcal{B}$ means that argument $\mathcal{A}$ attacks argument $\mathcal{B}$. Semantics to an AF can be given by labellings [7]. A labelling L is a function $L : \mathsf{Arg} \rightarrow \{\mathsf{in}, \mathsf{out}, \mathsf{undec}\}$ that evaluates a framework by stating whether an argument is accepted (in), rejected (out), or whether its status is not determined (undec). Further constraints, such as conflict-freeness and completeness, can be imposed on a labelling to obtain different sets of accepted arguments, cf. [1, 7].

Recently there has been interest in augmenting abstract argumentation with a probabilistic assignment to each argument [4, 6, 2]. Here, we regard the assignment as denoting the belief that an agent has that an argument is justifiable, i. e., that both the premises of the argument and the derivation of the claim of the argument from its premises are valid. So for a probability function P, and an argument $\mathcal{A}$, $P(\mathcal{A}) > 0.5$ denotes that the argument is believed (to the degree given by $P(\mathcal{A})$), $P(\mathcal{A}) < 0.5$ denotes that the argument is disbelieved (to the degree given by $P(\mathcal{A})$), and $P(\mathcal{A}) = 0.5$ denotes that the argument is neither believed or disbelieved. More precisely, a *probability function* P on Arg is a function $P : 2^{\mathsf{Arg}} \rightarrow [0, 1]$ with $\sum_{E \subseteq \mathsf{Arg}} P(E) = 1$. We abbreviate $P(\mathcal{A}) = \sum_{\mathcal{A} \in E \subseteq \mathsf{Arg}} P(E)$. This means that the probability of an argument is the sum of the probabilities of all sets of arguments that contain that argument.

The framework that we present in this paper is appealing theoretically as it provides further insights into semantics for abstract argumentation, and it offers a finer-grained representation of uncertainty in arguments. However, given an argument graph, it may be difficult for a user to assign a value to every argument. The user might have knowledge in order to identify a value for some arguments, but the user may be unable or unwilling to make assignments to the remaining arguments. This means that the user can only provide a partial assignment. If this is the case, then it would be desirable to have techniques to handle this incomplete information. In this short paper, we take a first step towards this direction and investigate the issue of completing incomplete probability assignments.

2 Constraints on probability functions

We first consider some constraints on the probability function which may take different aspects of the structure of the argument graph into account.

COH P is *coherent* wrt. AF if for every $\mathcal{A}, \mathcal{B} \in \mathsf{Arg}$, if $\mathcal{A} \rightarrow \mathcal{B}$ then $P(\mathcal{A}) \leq 1 - P(\mathcal{B})$.
SFOU P is *semi-founded* wrt. AF if $P(\mathcal{A}) \geq 0.5$ for every unattacked $\mathcal{A} \in \mathsf{Arg}$.
FOU P is *founded* wrt. AF if $P(\mathcal{A}) = 1$ for every unattacked $\mathcal{A} \in \mathsf{Arg}$.
SOPT P is *semi-optimistic* wrt. AF if $P(\mathcal{A}) \geq 1 - \sum_{\mathcal{B} \rightarrow \mathcal{A}} P(\mathcal{B})$ for every $\mathcal{A} \in \mathsf{Arg}$ that has at least one attacker.
OPT P is *optimistic* wrt. AF if $P(\mathcal{A}) \geq 1 - \sum_{\mathcal{B} \rightarrow \mathcal{A}} P(\mathcal{B})$ for every $\mathcal{A} \in \mathsf{Arg}$.
JUS P is *justifiable* wrt. AF if P is coherent and optimistic.
TER P is *ternary* wrt. AF if $P(\mathcal{A}) \in \{0, 0.5, 1\}$ for every $\mathcal{A} \in \mathsf{Arg}$.
RAT P is *rational* wrt. AF if for every $\mathcal{A}, \mathcal{B} \in \mathsf{Arg}$, if $\mathcal{A} \rightarrow \mathcal{B}$ then $P(\mathcal{A}) > 0.5$ implies $P(\mathcal{B}) \leq 0.5$.
NEU P is *neutral* wrt. AF if $P(\mathcal{A}) = 0.5$ for every $\mathcal{A} \in \mathsf{Arg}$.
INV P is *involutary* wrt. AF if for every $\mathcal{A}, \mathcal{B} \in \mathsf{Arg}$, if $\mathcal{A} \rightarrow \mathcal{B}$, then $P(\mathcal{A}) = 1 - P(\mathcal{B})$.
MAX P is *maximal* wrt. AF if $P(\mathcal{A}) = 1$ for every $\mathcal{A} \in \mathsf{Arg}$.
MIN P is *minimal* wrt. AF if $P(\mathcal{A}) = 0$ for every $\mathcal{A} \in \mathsf{Arg}$.

Let $\mathcal{P}(\mathsf{AF})$ be the set of all probability functions on Arg and $\mathcal{P}_t(\mathsf{AF})$ be the set of all t-probability functions where t is a constraint from above. We obtain[3] the strict classification of classes of probability functions as depicted in Figure 1.

3 Partial probability functions

A partial function $\pi : \mathsf{Arg} \rightarrow [0, 1]$ on Arg is called a *partial probability assignment*. A probability function $P \in \mathcal{P}(\mathsf{Arg})$ is π-*compliant* if for every $\mathcal{A} \in \mathsf{dom}\,\pi$ we have $\pi(\mathcal{A}) = P(\mathcal{A})$. Let $\mathcal{P}^{\pi}(\mathsf{AF}) \subseteq \mathcal{P}(\mathsf{AF})$ be the set of all π-compliant probability functions. The question that arises is that given an abstract argumentation framework $\mathsf{AF} = (\mathsf{Arg}, \rightarrow)$ and a partial probability assignment π, how do we determine $P \in \mathcal{P}(\mathsf{Arg})$ that is most compatible with both AF and π, i. e., which $P \in \mathcal{P}(\mathsf{Arg})$ do we select as a meaningful representative? This question has also been addressed in similar ways

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[3] For proofs of technical results see the extended version of this paper [3].

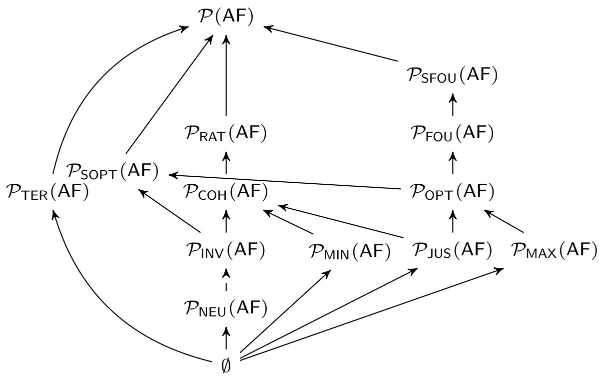


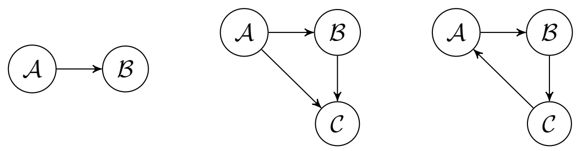
Figure 1. Classes of probability functions (a normal arrow $\rightarrow$ indicates a strict subset relation, a dashed arrow $\dashrightarrow$ indicates a subset relation)

for partial probabilistic information without argumentation, cf. e. g. [5]. There, the principle of *maximum entropy* has been used to complete incomplete probabilistic information in probabilistic logics. An important requirement for applying maximum entropy approaches is that the probability function with maximum entropy is uniquely determined. A sufficient property to ensure this, is that the set under consideration is both *convex* and *closed*.[4]

Proposition 1. *Let* $\mathsf{AF} = (\mathsf{Arg}, \rightarrow)$ *be an abstract argumentation framework. The sets* $\mathcal{P}(\mathsf{AF})$, $\mathcal{P}_{\mathsf{COH}}(\mathsf{AF})$, $\mathcal{P}_{\mathsf{TER}}(\mathsf{AF})$, $\mathcal{P}_{\mathsf{NEU}}(\mathsf{AF})$, $\mathcal{P}_{\mathsf{INV}}(\mathsf{AF})$, $\mathcal{P}_{\mathsf{SFOU}}(\mathsf{AF})$, $\mathcal{P}_{\mathsf{FOU}}(\mathsf{AF})$, $\mathcal{P}_{\mathsf{OPT}}(\mathsf{AF})$, $\mathcal{P}_{\mathsf{SOPT}}(\mathsf{AF})$, $\mathcal{P}_{\mathsf{JUS}}(\mathsf{AF})$, $\mathcal{P}_{\mathsf{MIN}}(\mathsf{AF})$, *and* $\mathcal{P}_{\mathsf{MAX}}(\mathsf{AF})$ *are convex and closed.*

Proposition 2. *For every partial probability assignment* π *the set* $\mathcal{P}^{\pi}(\mathsf{AF})$ *is convex and closed.*

Let t be any one of our properties which lead to a convex and closed set of probability functions (or any combination of those). If it is the case that there is at least one π-compliant P in $\mathcal{P}_t(\mathsf{AF})$ then (thanks to the convexity properties) we have that the intersection of $\mathcal{P}^{\pi}(\mathsf{AF})$ and $\mathcal{P}_t(\mathsf{AF})$ is convex and closed as well, cf. [5]. In that case, we can select the probability function with maximal entropy within this intersection (which is uniquely defined). As for the rationale of this decision, several results from probability reasoning, as for example discussed in [5], can be harnessed. We continue with



(a) (b) (c)

Figure 2. Argumentation frameworks for Example 1

some examples to illustrate the definitions and to investigate some of our concerns in dealing with partial assignments.

[4] A set X is *convex* if for $x_1, x_2 \in X$ it also holds that $\delta x_1 + (1-\delta)x_2 \in X$ for every $\delta \in [0,1]$; a set X is closed if for every converging sequence $x_1, x_2, \ldots$ with $x_i \in X$ ($i \in \mathbb{N}$) we have that $\lim_{i\rightarrow\infty} x_i \in X$

Example 1. *For the argumentation framework depicted in Figure 2(a) consider* π_1 *with* $\pi_1(\mathcal{A}) = 1$. *Obviously, the most reasonable choice for a* π_1*-compliant* $P \in \mathcal{P}(\mathsf{AF})$ *would be* $P(\mathcal{A}) = 1$ *and* $P(\mathcal{B}) = 0$ *(by obeying the property of involution). Furthermore, for* $\pi_2(\mathcal{B}) = 0.3$ *we would have* $P(\mathcal{A}) = 0.7$ *and* $P(\mathcal{B}) = 0.3$ *following the same rationale.*

For the argumentation framework depicted in Figure 2(b) consider π_3 *with* $\pi_3(\mathcal{C}) = 0.4$. *A possible choice for* P *would be* $P(\mathcal{A}) = 0.6$, $P(\mathcal{B}) = 0.4$, *and* $P(\mathcal{C}) = 0.4$ *(having thus a maximally committed function that is coherent). But note that the set* $\mathcal{P}^{\pi}(\mathsf{AF}) \cap \mathcal{P}_{COH}(\mathsf{AF})$ *does contain more than this single probability function. Furthermore, for* π_4 *with* $\pi_4(\mathcal{B}) = 0.7$ *and* $\pi_4(\mathcal{C}) = 0.6$ *one would only guess* $P(\mathcal{A}) \leq 0.3$ *but due to the "inconsistency" of* π_4 *(violating the coherence condition), what is the best choice?*

For the argumentation framework depicted in Figure 2(c) consider π_5 *with* $\pi_5(\mathcal{A}) = 0.4$ *and the following four selections* $P_1, P_2, P_3, P_4 \in \mathcal{P}(\mathsf{AF})$:

$$
\begin{array}{cccc}
P_1(\mathcal{A}) = 0.4 & P_2(\mathcal{A}) = 0.4 & P_3(\mathcal{A}) = 0.4 & P_4(\mathcal{A}) = 0.4 \\
P_1(\mathcal{B}) = 0.6 & P_2(\mathcal{B}) = 0.4 & P_3(\mathcal{B}) = 0.5 & P_4(\mathcal{B}) = 0.2 \\
P_1(\mathcal{C}) = 0.4 & P_2(\mathcal{C}) = 0.6 & P_3(\mathcal{C}) = 0.5 & P_4(\mathcal{C}) = 0.3
\end{array}
$$

All of the above probability functions are π_5*-compliant and coherent. Function* P_4 *is not maximally committed and as such is perhaps not a good choice. Both* P_1 *and* P_2 *are "extreme points of view" and model some kind of probabilistic stable semantics. The function* P_3 *is as unbiased as possible but still "reasonable" as it models probabilistic grounded semantics. Note that* P_3 *is also the probability function with maximal entropy in* $\mathcal{P}^{\pi}(\mathsf{AF}) \cap \mathcal{P}_{COH}(\mathsf{AF})$.

Given $\mathcal{P}^{\pi}(\mathsf{AF})$ and $\mathcal{P}_t(\mathsf{AF})$, we can either select $P \in \mathcal{P}^{\pi}(\mathsf{AF})$ that is "as close as possible to" $\mathcal{P}_t(\mathsf{AF})$ or $P \in \mathcal{P}_t(\mathsf{AF})$ that is "as close as possible to" $\mathcal{P}^{\pi}(\mathsf{AF})$. In future work, we will investigate definitions for "as close as possible to", and we will explore the pros and cons of each of these alternatives for selecting P.

4 Summary

In this paper, we discussed several constraints for probabilistic abstract argumentation and applied this framework to the problem of completing partial probability assignments. A first investigation leads us to believe that maximizing entropy within probability functions of a specific type gives appropriate results for this problem.

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