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Aggregating Ranking-based Semantics in Abstract Argumentation

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Abstract. We investigate how multiple ranking-based semantics can be aggregated into a single ranking-based semantics for abstract argumentation frameworks. For that, we phrase this problem as a judgment aggregation problem and utilise ideas from voting theory for aggregating preorders. We show, that in our approach, the relationship between two arguments is preserved in cases where all ranking-based semantics agree on the relationship. We analyse the resulting ranking-based semantics axiomatically and demonstrate that the satisfaction of the principles can be transferred to the aggregated setting when all, or at least a majority, of the ranking-based semantics satisfy the principles.

Keywords. Abstract Argumentation, Ranking Semantics, Voting, Aggregation

1. Introduction

Formal argumentation [1] is concerned with models of rational decision-making based on representations of arguments and their relations. A particularly important and simple approach is that of abstract argumentation frameworks (AFs) [2], which represent argumentative scenarios as directed graphs. Here, *arguments* are identified by vertices, and an *attack* from one argument to another is represented as a directed edge. To reason over AFs, *ranking-based semantics* (and the related *gradual semantics*) were proposed [3,4,5], where arguments are ranked according to their individual strength.

In the literature, we can find a number of different approaches for ranking-based semantics [5], some of which utilise the topological structure of the AF [3], while others generalise the extension-based perspective to rankings [6,7]. All of these approaches have been analysed axiomatically, thus they can be differentiated based on the principles they satisfy and violate. We refer the reader to [8,5] for more details. Two semantics are equivalent if they satisfy the same set of principles. However, Amgoud et al. [8] have shown that satisfying the same principles does not guarantee that the induced rankings will be the same. Thus, different ranking-based semantics can be seen as points of view of agents. To obtain a collective view across the different agents, we aggregate the individual rankings. The goal of this paper is to formally investigate how to aggregate different ranking-based semantics.

In order to achieve an aggregation of rankings, we adapt approaches from *voting theory* [9], an area of research focusing on collective decision-making within a group of individuals. We exemplify the ideas by the concrete examples of the following rules *plurality rule*, *Borda rule*, and *Copeland rule* to aggregate the individual rankings. We consider the ranking-based semantics as voters in an election and the arguments are the alternatives to be voted on.

We introduce the notion of *aggregated ranking-based semantics*, which takes a set of ranking-based semantics as input and returns one aggregated ranking. We then investigate the behaviour of these semantics in detail. One of our results is that the relationship between the arguments is in most cases *preserved*, meaning that if the relationship between two arguments coincides in all individual rankings, then the relationship between these two arguments should be the same in the aggregated case.We also analyse the newly introduced ranking-based semantics axiomatically. We prove that satisfaction of principles can be *transferred* from the individual ranking to the aggregated one if all, or at-least the majority, of the ranking-based semantics satisfy these principles.

The structure of this paper is as follows: In Section 2, we recall the necessary background information on AFs and ranking-based semantics. We formally introduce aggregated ranking-based semantics in Section 3 and discuss three instantiations. Section 4 contains a discussion about the preservation of the relationships of the aggregated rankings. In Section 5, we show that some principles for ranking-based semantics can be satisfied by the aggregated ranking-based semantics if all or at-least the majority of the to-be-aggregated semantics satisfy that particular principle. We conclude the paper in Section 6 by discussing related work and future work avenues. All proofs can be found in the supplementary material[1].

2. Preliminaries

An *abstract argumentation framework* (*AF*) is a directed graph $F = (A, R)$, where A is a (finite) set of *arguments* and $R \subseteq A \times A$ is an *attack relation* among them [2]. An argument a is said to *attack* an argument b if $(a,b) \in R$. We say that an argument a is *defended by a set* $E \subseteq A$ if every argument $b \in A$ that attacks a is attacked by some $c \in E$. For $a \in A$, we define $a_F^- = \{b \mid (b,a) \in R\}$ and $a_F^+ = \{b \mid (a,b) \in R\}$ as the sets of arguments attacking a and the sets of arguments that are attacked by a in $F = (A,R)$. For a set of arguments $E \subseteq A$, we extend these definitions to E_F^- and E_F^+ via $E_F^- = \bigcup_{a \in E} a_F^-$ and $E_F^+ = \bigcup_{a \in E} a_F^+$, respectively. If the AF is clear in the context, we will omit the index. For two AFs $F = (A,R)$ and $F' = (A',R')$ we denote with $F \oplus F' = (A \cup A', R \cup R')$ the union between F and F'. Most semantics [10] for abstract argumentation rely on two basic concepts: *conflict-freeness* and *admissibility*.

Definition 1. *Given* $F = (A,R)$*, a set* $E \subseteq A$ *is:* conflict-free *iff* $\forall a,b \in E$, $(a,b) \notin R$*;* admissible *iff it is conflict-free, and every element of* E *is defended by* E*.*

For an AF F, we use $cf(F)$ and $ad(F)$ to denote the sets of conflict-free and admissible sets, respectively. We define the remaining semantics (denoted as *extension semantics*) as proposed in [2].

Definition 2. *For an AF* $F = (A,R)$ *and a set of arguments* $E \subseteq A$*. An admissible set* $E \subseteq A$ *is: a* complete *extension (co) iff it contains every defended argument; a* preferred

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[1] `https://e.feu.de/26t`

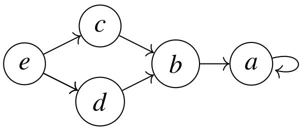

Figure 1. AF F_1 from Example 1.

extension (pr) iff it is a $\subseteq$-maximal complete extension; the unique grounded *extension (gr) iff it is the $\subseteq$-minimal complete extension; a* stable *extension (stb) iff $E_F^+ = A \setminus E$. With $\sigma(F)$ for $\sigma \in \{co, pr, gr, stb\}$, we denote the set of σ-extensions.*

We call an argument a, that is part of at least one σ-extension of AF F, *credulously accepted* with respect to extension semantics σ in F ($a \in \mathsf{cred}_\sigma(F)$), otherwise if a is not part of any σ-extension of F it is called *rejected* with respect to σ in F ($a \in \mathsf{rej}_\sigma(F)$). An argument a is *skeptically accepted* with respect to extensions semantics σ in AF F, if a is part of every σ extension of F ($a \in \mathsf{skep}_\sigma(F)$).

Example 1. *Consider the AF $F_1 = (A, R)$ depicted in Figure 1 as a directed graph. F_1 has a unique complete extension: $\{e, b\}$. Thus, e and b are the only credulously and skeptically accepted arguments with respect to the complete extension.*

Instead of reasoning based on the acceptance of sets of arguments, *gradual semantics* and *ranking-based semantics* [3] are ranking the individual arguments based on their strength. See [11,5] for overviews of this topic.

Definition 3. *A* ranking-based semantics *ρ is a function which maps an AF $F = (A, R)$ to a preorder*[2] *$\succeq_F^\rho$ on A.*

Intuitively $a \succeq_F^\rho b$ means that a is at least as strong as b in F. We define the usual abbreviations as follows; $a \succ_F^\rho b$ denotes *strictly stronger*, i. e. $a \succeq_F^\rho b$ and $b \not\succeq_F^\rho a$. $a =_F^\rho b$ denotes *equally strong*, i. e. $a \succeq_F^\rho b$ and $b \succeq_F^\rho a$.

For ranking-based semantics, we find a number of principles to evaluate and compare the different approaches. We present a short list of principles relevant to this paper. For a longer discussion on the principles of the ranking-based semantics, we refer to [4].

Definition 4. *A ranking-based semantics ρ satisfies the respective principle iff for all AFs $F = (A, R)$ and any $a, b \in A$:*

Abstraction (Abs). *The names of the arguments do not affect the ranking.*
For a pair of AFs $F = (A, R)$ and $F' = (A', R')$ and every isomorphism $\gamma : A \to A'$, we have $a \succeq_F^\rho b$ if and only if $\gamma(a) \succeq_{F'}^\rho \gamma(b)$.

Independence (In). *Unconnected arguments should not influence a ranking.*
For every $F' = (A', R') \in cc(F)$ and for all $a, b \in A'$: $a \succeq_F^\rho b$ if and only if $a \succeq_{F'}^\rho b$, where $cc(F)$ are the connected components *of F.*

Non-attacked Equivalence (NaE). *Two unattacked arguments should be equally ranked. If $a_F^- = b_F^- = \emptyset$, then $a =_F^\rho b$.*

Void Precedence (VP). *Unattacked arguments should be ranked better than attacked ones: If $a_F^- = \emptyset$ and $b_F^- \neq \emptyset$ then $a \succ_F^\rho b$.*

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[2] A preorder is a (binary) relation that is *reflexive* and *transitive*.

Self-Contradiction (SC). *Self-attacking arguments should be ranked worse than any other argument: If* $(a,a) \notin R$ *and* $(b,b) \in R$ *then* $a \succ_F^{\rho} b$.

Cardinality Precedence (CP). *An argument with less attackers is ranked better than one with more attackers: If* $|a_F^-| < |b_F^-|$ *then* $a \succ_F^{\rho} b$.

Quality Precedence (QP). *If there is an attacker of an argument which is ranked better than every attacker of an other argument, the latter is ranked better than the former: If there is* $c \in b_F^-$ *such that for all* $d \in a_F^-$ *it holds that* $c \succ_F^{\rho} d$ *then* $a \succ_F^{\rho} b$.

Counter-Transitivity (CT). *If the attackers of an argument are as numerous and as strong as the attackers of another argument, then the former is as least as strong as the latter.*
If some injective $f : a_F^- \to b_F^-$ *exists such that* $f(x) \succeq_F^{\rho} x$ *for all* $x \in a_F^-$, *then* $a \succeq_F^{\rho} b$.

Strict Counter-Transitivity (SCT). *Strict version of CT.*
If some injective $f : a_F^- \to b_F^-$ *exists such that* $f(x) \succeq_F^{\rho} x$ *for all* $x \in a_F^-$ *and either* $|a_F^-| < |b_F^-|$ *or there exists some* $x \in a_F^-$ *with* $f(x) \succ_F^{\rho} x$, *then* $a \succ_F^{\rho} b$.

Attack vs. Full Defense (AvsFD). *Arguments with no unattacked indirect attackers should be ranked better than arguments attacked only by one unattacked argument. If* F *is acyclic and every path* $P(u,a)$ *in* F *from an unattacked argument* u *to* a *has an even length* $l_p = 0 \bmod 2$ *and there exist unattacked* $v \in b_F^-$, *then* $a \succ_F^{\rho} b$.

σ-Compatibility (σ-C). *Credulously accepted arguments should be ranked better than rejected arguments: For an extension semantics* σ *it holds that if* $a \in \mathsf{cred}_\sigma(F)$ *and* $b \in \mathsf{rej}_\sigma(F)$, *then* $a \succ_F^{\rho} b$.

In this paper, we consider the ranking-based semantics *discussion-based ranking-based semantics* [3], *h-categoriser ranking-based semantics* [12] and *strategy-based ranking-based semantics* [13] as examples.

The *discussion-based ranking-based semantics* [3] compares two arguments with respect to the number of attackers or defenders they have.

Definition 5. *Let* $F = (A,R)$ *be an AF. The* discussion count $\mathsf{dis}(a)$ *of argument* $a \in A$ *is a vector* $\mathsf{dis}(a) = \langle \mathsf{dis}(a)_1, \mathsf{dis}(a)_2, \dots \rangle$, *where* $\mathsf{dis}_i(a)$ *for* $i \in \{0, \dots, |A|\}$ *is defined via:*

$$\mathsf{dis}_i(a) := \begin{cases} -|a_i^-| & \text{if } i \text{ is odd} \\ |a_i^-| & \text{if } i \text{ is even} \end{cases}$$

Where a_i^- *denotes the set of arguments with a walk to* a *of length* i. *So, there is a sequence* $s = (a_0, \dots, a_n)$ *s.t. for all* $b \in a_i^-$ *with* $a_0 = b$, $a_n = a$ *and for all* $i < n$, $(a_i, a_{i+1}) \in R$.

The discussion-based ranking-based semantics *(*Dbs*) is defined via:*

$$a \succeq_F^{\mathsf{Dbs}} b \text{ if and only if } \mathsf{dis}(a) \geq^{lex} \mathsf{dis}(b)$$

Where $\geq^{lex}$ *is the* lexicographic bigger *relation.*

One example of a gradual semantics is the *h-categoriser ranking-based semantics* [12]. The h-categoriser ranking-based semantics considers the strength of the attackers to calculate the strength of an argument.

Definition 6. *Let* $F = (A,R)$ *be an AF and* $a \in A$. *A* h-categoriser function $\mathsf{cat} : A \to (0,1]$ *is a function that satisfies:*

$$\mathsf{cat}(a) = \frac{1}{1 + \sum_{b \in a_F^-} \mathsf{cat}(b)}$$

We define the h-categoriser ranking-based semantics *(*Cat*) as:*

$$a \succeq_F^{\mathsf{Cat}} b \text{ if and only if } \mathsf{cat}(a) \geq \mathsf{cat}(b)$$

It was shown that the h-categoriser ranking-based semantics is well-defined, this means that there is a unique h-categoriser function for each AF [14].

The final ranking-based semantics we are considering is the *strategy-based ranking-based semantics* proposed in [13]. This semantics utilises a two-player zero-sum game to rank arguments. For an AF $F = (A, R)$ and argument a, the set of pure strategies of the proponent is $S_P(a) = \{P \mid P \subseteq A, a \in P\}$ and for the opponent it is $S_O = \{O \mid O \subseteq A\}$.

Definition 7. *Let* $F = (A,R)$ *be an AF and* $P, O \subseteq A$. *The* set of attacks from P *to* O *is denoted by* $O_F^{\leftarrow P} = \{(a,b) \in P \times O \mid (a,b) \in R\}$.

The degree of plausibility *of* P *w.r.t.* O *is:* $\phi(P,O) = \frac{1}{2}[1 + f(|O_F^{\leftarrow P}|) - f(|P_F^{\leftarrow O}|)]$ *with* $f(x) = \frac{x}{x+1}$. *The corresponding* rewards of P *(*$r_F(P,O)$*) are:*

$$r_F(P,O) := \begin{cases} 0 & \textit{iff } \exists a,b \in P, (a,b) \in R, \\ 1 & \textit{iff } |P_F^{\leftarrow O}| = 0, \\ \phi(P,O) & \textit{otherwise.} \end{cases}$$

Let $p = (p_1, p_2, \ldots, p_{|S_P|})$ *be a mixed strategy of the proponent and* $q = (q_1, q_2, \ldots, q_{|S_O|})$ *a mixed strategy of the opponent with probabilities* p_i *and* q_i. *For each* $a \in A$, *the proponent's* expected payoff *is:* $E(a,p,q) = \sum_{j=1}^{|S_O|} \sum_{i=1}^{|S_P|} p_i q_j r_{i,j}$ *for* $r_{i,j}$ *being the ith and jth (pure) strategy of the proponent resp. opponent. The value of the zero-sum game for argument* a *is:* $s(a) = \max_p \min_q E(a,p,q)$. *The* strategy-based ranking-based semantics *(*$\mathsf{2ZG}$*) is defined via:* $a \succeq_F^{\mathsf{2ZG}} b$ *if and only if* $s(a) \geq s(b)$

Example 2. *Consider again* F_1 *from Example 1. The argument rankings induced by the previously introduced ranking-based semantics are:*

$$\mathsf{Dbs}: e \succ_{F_1}^{\mathsf{Dbs}} d =_{F_1}^{\mathsf{Dbs}} c \succ_{F_1}^{\mathsf{Dbs}} a \succ_{F_1}^{\mathsf{Dbs}} b \qquad \mathsf{Cat}: e \succ_{F_1}^{\mathsf{Cat}} d =_{F_1}^{\mathsf{Cat}} c =_{F_1}^{\mathsf{Cat}} a =_{F_1}^{\mathsf{Cat}} b$$

$$\mathsf{2ZG}: e \succ_{F_1}^{\mathsf{2ZG}} b \succ_{F_1}^{\mathsf{2ZG}} c =_{F_1}^{\mathsf{2ZG}} d \succ_{F_1}^{\mathsf{2ZG}} a$$

Cat *only differentiates* e *from the other arguments and ranks the remaining arguments as equally strong. Semantics* $\mathsf{2ZG}$ *differs most from the other rankings. While all of them agree that* e *is the strongest argument, only* $\mathsf{2ZG}$ *ranks the self-attacking argument* a *as the weakest.*

The satisfied and violated principles of the three introduced ranking-based semantics are summarised later in Table 2 (left side).

3. Aggregating Ranking-based Semantics

Already in Example 2, we have seen that ranking-based semantics induce different rankings for the same AF. We can interpret the different semantics as views of agents on the strength of arguments. The agent based on $\mathsf{2ZG}$ argues that argument a is the weakest argument, while the other two agents would disagree. The agent based on Dbs would even argue that b is strictly weaker than a. While the three agents disagree on the weakest argument, they agree at least that argument e is the strongest. Thus, in their collective view, they agree that argument e is the strongest. In the following, we propose approaches to model the collective view of the three agents. We start by introducing *aggregated ranking-based semantics*, which maps different views of agents (represented by ranking-based semantics) to an aggregated view within an AF.

Definition 8. *An* aggregator Θ *maps a multiset of ranking-based semantics* $\mathscr{P}$ *to a ranking-based semantics* $\Theta(\mathscr{P})$, *which we call* aggregated ranking-based semantics.

A variety of different aggregation methods are imaginable. In this work, we focus on approaches from the area of *voting theory* [9], where methods and processes for collective decision-making are explored within a group of individuals.

3.1. Plurality-based ranking-based semantics

The most fundamental approach to facilitate an election is the *plurality rule*, in which for every voter only the most preferred alternative is considered. If an alternative is positioned in the first position by a voter, they receive one point, otherwise zero points are awarded. The alternative with the highest total number of points is then considered the winner. Note that in our context, we consider preorders and thus the most preferred alternative is not necessarily unique.

Definition 9. *Let* $\succeq$ *be a preorder on a set* A*. An element* $a \in A$ *is considered* maximal *wrt.* $\succeq$ *if there is no element* $b \in A$ *s.t.* $b \succ a$*.*
With $\text{Max}_{\succeq}(A)$*, we denote the maximal elements of* A *wrt. preorder* $\succeq$*.*

Next, we define the *plurality-based ranking-based semantics*.

Definition 10. *Let* $F = (A,R)$ *be an AF and* $\mathscr{P} = \{\rho_1, \ldots, \rho_n\}$ *a multiset of ranking-based semantics. The* priorityfunction $\text{Prio}_{\succeq^{\rho}_{F}} : A \rightarrow [0,1]$ *assigns every maximal ranked argument w.r.t* $\rho \in \mathscr{P}$ *the value of 1 and 0 to the other arguments.*

$$\text{Prio}_{\succeq^{\rho}_{F}}(a) = \begin{cases} 1, & \textit{if } a \in \text{Max}_{\succeq^{\rho}_{F}}(A), \\ 0, & \textit{else.} \end{cases}$$

We aggregate the values each argument receives from each preorder by summation. The plurality-based ranking-based semantics $\mathsf{pl}(\mathscr{P})$ *is defined via:*

$$a \succeq^{\mathsf{pl}(\mathscr{P})}_{F} b \textit{ iff } \textstyle\sum_{i=1}^{n} \text{Prio}_{\succeq^{\rho_i}_{F}}(a) \geq \sum_{i=1}^{n} \text{Prio}_{\succeq^{\rho_i}_{F}}(b)$$

An argument, which is ranked as the best argument according to the most ranking-based semantics, is the best ranked argument in the aggregated setting.

Example 3. *We continue Example 2. Let* $\mathscr{P}_1 = \{\mathsf{Dbs}, \mathsf{Cat}, \mathsf{2ZG}\}$ *be a multiset of ranking-based semantics. In all three rankings from* $\mathscr{P}_1$ *argument* e *is the maximal ranked argument, while all other arguments are ranked below* e*. Thus, we get:* $\text{Prio}_{\succeq^{\rho}_{F_1}}(e) = 1$ *and* $\text{Prio}_{\succeq^{\rho}_{F_1}}(a) = \text{Prio}_{\succeq^{\rho}_{F_1}}(b) = \text{Prio}_{\succeq^{\rho}_{F_1}}(c) = \text{Prio}_{\succeq^{\rho}_{F_1}}(d) = 0$ *for all* $\rho \in \mathscr{P}_1$*. The aggregated ranking is then:*

$$e \succ^{\mathsf{pl}(\mathscr{P}_1)}_{F_1} a =^{\mathsf{pl}(\mathscr{P}_1)}_{F_1} b =^{\mathsf{pl}(\mathscr{P}_1)}_{F_1} c =^{\mathsf{pl}(\mathscr{P}_1)}_{F_1} d.$$

3.2. Borda$^{\alpha}$*-based ranking-based semantics*

A more involved approach than the plurality rule for elections is the *Borda rule*. Here, alternatives receive points according to their position in a preference order. The most preferred alternatives receive the maximal amount of points, the alternatives in the second position receive one point less and so on. The aggregated preorder over the alternatives is then the summation of all received points. Unlike in voting theory, in our context we have preorders, thus it is necessary to define how to handle incomparable arguments. We do not award points based on the position in the preorder, instead an argument receives one point for every argument it is strictly stronger than. Two equally strong arguments receive $\alpha \in [0,1]$ points. Thus, we define a family of *Borda*$^{\alpha}$*-based ranking-based semantics*.

Definition 11. *Let $F=(A,R)$ be an AF, $\mathscr{P}=\{\rho_1,\dots,\rho_n\}$ a multiset of ranking-based semantics and $\alpha\in[0,1]$. The* Borda$^\alpha$-score $\mathsf{Borda}^{\alpha}_{\mathscr{P},F}: A\rightarrow\mathbb{R}$ *for argument $a\in A$ is defined via:*

$$\mathsf{Borda}^{\alpha}_{\mathscr{P},F}(a)=\sum_{i=1}^{n}\left(\left|\{b\in A\mid a\succ^{\rho_i}_{F} b\}\right|+\alpha\left|\{b\in A\setminus\{a\}\mid a=^{\rho_i}_{F} b\}\right|\right)$$

The corresponding Borda$^\alpha$-based ranking-based semantics $\mathsf{B}^{\alpha}(\mathscr{P})$ *is defined as:*

$$a\succeq^{\mathsf{B}^{\alpha}(\mathscr{P})}_{F} b \text{ iff } \mathsf{Borda}^{\alpha}_{\mathscr{P},F}(a)\geq\mathsf{Borda}^{\alpha}_{\mathscr{P},F}(b)$$

Argument a is ranked higher than argument b if there are more arguments for which a is ranked higher than there are arguments for which b is ranked higher.

Example 4. *Consider again F_1 and the corresponding argument rankings from Example 2. The Borda$^\alpha$-scores of the arguments are:*

$$\mathsf{Borda}^{\alpha}_{\mathscr{P}_1,F_1}(a)=1+3\alpha,\qquad \mathsf{Borda}^{\alpha}_{\mathscr{P}_1,F_1}(b)=3+3\alpha,$$
$$\mathsf{Borda}^{\alpha}_{\mathscr{P}_1,F_1}(c)=\mathsf{Borda}^{\alpha}_{\mathscr{P}_1,F_1}(d)=3+5\alpha,\qquad \mathsf{Borda}^{\alpha}_{\mathscr{P}_1,F_1}(e)=12.$$

The Borda$^\alpha$-based rankings for $\alpha=0$ and $\alpha\in(0,1]$ are:

$$e\succ^{\mathsf{B}^{0}(\mathscr{P}_1)}_{F_1} c=^{\mathsf{B}^{0}(\mathscr{P}_1)}_{F_1} d=^{\mathsf{B}^{0}(\mathscr{P}_1)}_{F_1} b\succ^{\mathsf{B}^{0}(\mathscr{P}_1)}_{F_1} a,$$
$$e\succ^{\mathsf{B}^{\alpha}(\mathscr{P}_1)}_{F_1} c=^{\mathsf{B}^{\alpha}(\mathscr{P}_1)}_{F_1} d\succ^{\mathsf{B}^{\alpha}(\mathscr{P}_1)}_{F_1} b\succ^{\mathsf{B}^{\alpha}(\mathscr{P}_1)}_{F_1} a.$$

3.3. Copeland$^\alpha$-based ranking-based semantics

The final voting rule we are discussing is the *Copeland rule*. This approach is based on pairwise comparisons of the alternatives according to the individual preferences of the voters. For each pair of alternatives a and b, we check how many voters prefer a to b. For each strict win, a receives one point and for each tie, a receives $\alpha\in[0,1]$ points[3]. We consider a general family of *Copeland$^\alpha$-based ranking-based semantics* for $\alpha\in[0,1]$. Arguments with the most wins in pairwise comparisons are the best ranked ones.

Definition 12. *Let $F=(A,R)$ be an AF, $\mathscr{P}=\{\rho_1,\dots,\rho_n\}$ a multiset of ranking-based semantics and $\alpha\in[0,1]$. The* pairwise majority-relation *for $a,b\in A$ is defined by:*

$$a>^{\mu}_{\mathscr{P},F} b \quad\text{iff}\quad \left|\{\rho_i\in\mathscr{P}: a\succeq^{\rho_i}_{F} b\}\right|>\left|\{\rho_i\in\mathscr{P}: b\succeq^{\rho_i}_{F} a\}\right|$$
$$a=^{\mu}_{\mathscr{P},F} b \quad\text{iff}\quad \left|\{\rho_i\in\mathscr{P}: a\succeq^{\rho_i}_{F} b\}\right|=\left|\{\rho_i\in\mathscr{P}: b\succeq^{\rho_i}_{F} a\}\right|$$

The Copeland$^\alpha$-score $\mathsf{Copeland}^{\alpha}_{\mathscr{P},F}: A\rightarrow\mathbb{N}$ *for argument $a\in A$ is defined via:*

$$\mathsf{Copeland}^{\alpha}_{\mathscr{P},F}(a)=\left|\{b\in A\mid a>^{\mu}_{\mathscr{P},F} b\}\right|+\alpha\left|\{b\in A\setminus\{a\}\mid a=^{\mu}_{\mathscr{P},F} b\}\right|$$

The corresponding Copeland$^\alpha$-based ranking-based semantics $\mathsf{C}^{\alpha}(\mathscr{P})$ *is defined as:*

$$a\succeq^{\mathsf{C}^{\alpha}(\mathscr{P})}_{F} b \text{ iff } \mathsf{Copeland}^{\alpha}_{\mathscr{P},F}(a)\geq\mathsf{Copeland}^{\alpha}_{\mathscr{P},F}(b)$$

Example 5. *Consider again F_1 and the corresponding argument rankings from Example 2. The pairwise comparisons are depicted in Table 1. Hence, the Copeland$^\alpha$-scores are:*

$$\mathsf{Copeland}^{\alpha}_{\mathscr{P}_1,F_1}(a)=\alpha,\qquad \mathsf{Copeland}^{\alpha}_{\mathscr{P}_1,F_1}(b)=3\alpha,$$
$$\mathsf{Copeland}^{\alpha}_{\mathscr{P}_1,F_1}(c)=\mathsf{Copeland}^{\alpha}_{\mathscr{P}_1,F_1}(d)=1+2\alpha,\qquad \mathsf{Copeland}^{\alpha}_{\mathscr{P}_1,F_1}(e)=4.$$

For $\alpha=0$, $\alpha\in(0,1)$ and $\alpha=1$, the corresponding Copeland$^\alpha$-based rankings are:

$$e\succ^{\mathsf{C}^{0}(\mathscr{P}_1)}_{F_1} c=^{\mathsf{C}^{0}(\mathscr{P}_1)}_{F_1} d\succ^{\mathsf{C}^{0}(\mathscr{P}_1)}_{F_1} b=^{\mathsf{C}^{0}(\mathscr{P}_1)}_{F_1} a,$$
$$e\succ^{\mathsf{C}^{\alpha}(\mathscr{P}_1)}_{F_1} c=^{\mathsf{C}^{\alpha}(\mathscr{P}_1)}_{F_1} d\succ^{\mathsf{C}^{\alpha}(\mathscr{P}_1)}_{F_1} b\succ^{\mathsf{C}^{\alpha}(\mathscr{P}_1)}_{F_1} a,$$
$$e\succ^{\mathsf{C}^{1}(\mathscr{P}_1)}_{F_1} c=^{\mathsf{C}^{1}(\mathscr{P}_1)}_{F_1} d=^{\mathsf{C}^{1}(\mathscr{P}_1)}_{F_1} b\succ^{\mathsf{C}^{1}(\mathscr{P}_1)}_{F_1} a.$$

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[3] This family of voting rules was introduced by Faliszewski et al. [15], while the traditional *Copeland rule* is the variant with $\alpha=\frac{1}{2}$. For $\alpha=1$ the corresponding voting rule is called *Llull's rule*.

	a vs b	a vs c	a vs d	a vs e	b vs c	b vs d	b vs e	c vs d	c vs e	d vs e
$\succeq_{F_1}^{\mathsf{Dbs}}$	a	c	d	e	c	d	e	c=d	e	e
$\succeq_{F_1}^{\mathsf{Cat}}$	a=b	a=c	a=d	e	b=c	b=d	e	c=d	e	e
$\succeq_{F_1}^{\mathsf{2ZG}}$	b	c	d	e	b	b	e	c=d	e	e
Winner	a=b	c	d	e	c=b	d=b	e	c=d	e	e

Table 1. Pairwise comparison for argument $\{a,b,c,d,e\}$ from F_1 for Example 5, where $x = y$ means that argument x and y are equally as strong.

4. Preservation of Rankings

In Examples 3–5, we saw that the three aggregated ranking-based semantics induce different argument rankings for the same AF. While the corresponding rankings have some similarities, for instance e is the strongest argument in all of them, they differ in their treatment of the relationship between arguments a and b. In the plurality-based and the Copeland^0-based rankings a and b are equally strong, in the Borda-based and Copeland$^\alpha$-based ones, for $\alpha \in (0,1]$, b is stronger than a. These differences in the relationship between a and b are already evident in the underlying argument rankings, which were aggregated. For Dbs, a is preferred over b, whereas for $\mathsf{2ZG}$ b is preferred over a. However, if all rankings agree on a relationship, for instance that c and d are equally strong, then the aggregated ranking-based semantics should reflect this behaviour.

Definition 13. *The aggregator* Θ *satisfies:*

- $\succeq$-preservation *iff* $a \succeq_F^\rho b$ *for all* $\rho \in \mathscr{P}$*, then also* $a \succeq_F^{\Theta(\mathscr{P})} b$ *holds for* $a,b \in A$*, for all AF* $F = (A,R)$*, for all multisets or ranking-based semantics* $\mathscr{P}$.
- $\succ$-preservation *iff* $a \succ_F^\rho b$ *for all* $\rho \in \mathscr{P}$*, then also* $a \succ_F^{\Theta(\mathscr{P})} b$ *holds for* $a,b \in A$*, for all AF* $F = (A,R)$*, for all multisets or ranking-based semantics* $\mathscr{P}$.
- $=$-preservation *iff* $a =_F^\rho b$ *for all* $\rho \in \mathscr{P}$*, then also* $a =_F^{\Theta(\mathscr{P})} b$ *holds for* $a,b \in A$*, for all AF* $F = (A,R)$*, for all multisets or ranking-based semantics* $\mathscr{P}$.

The preservation properties of the aggregated ranking-based semantics reflect the idea of *unanimity* from voting theory, thus if all voters agree on one alternative, this alternative should be the winner.

Proposition 1. *The following hold:*

- pl *satisfies* $\succeq$*-preservation and* $=$*-preservation.*
- B^α *satisfies* $\succeq$*-preservation,* $\succ$*-preservation, and* $=$*-preservation for all* $\alpha \in [0,1]$.
- C^α *satisfies* $\succeq$*-preservation,* $\succ$*-preservation, and* $=$*-preservation for all* $\alpha \in [0,1]$.

5. Axiomatic Investigation

All discussed aggregated ranking-based semantics satisfy some ranking preservation conditions. Hence, the aggregation operators induce reasonable preorders when viewed from the perspective of voting theory. However, the perspective of argumentation theory is missing. We start with showing that all aggregators induce a *total* preorder, regardless of the ranking-based semantics.

Proposition 2. *Let $\mathscr{P}$ be a multiset of ranking-based semantics and F be an AF.* $\succeq_F^{\mathsf{pl}(\mathscr{P})}$, $\succeq_F^{\mathsf{B}^\alpha(\mathscr{P})}$ *and* $\succeq_F^{\mathsf{C}^\alpha(\mathscr{P})}$ *are total preorders.*

5.1. Transferability

Similar to preserving the relationships between arguments, we investigate whether the satisfaction of principles can be preserved by aggregating ranking-based semantics. If all ranking-based semantics satisfy a principle P then their aggregation of them should satisfy P as well.

Definition 14. *A principle P is* transferable *by an aggregator Θ if the following holds: If all semantics $\rho \in \mathscr{P}$ satisfy P, then the aggregated ranking-based semantics $\Theta(\mathscr{P})$ satisfies P as well, for all multisets of ranking-based semantics $\mathscr{P}$.*

Not every principle is transferable, for instance In is not transferable for the aggregated ranking-based semantics discussed.

Example 6. *Let $\mathscr{P}_1 = \{\mathsf{Dbs}, \mathsf{Cat}, \mathsf{2ZG}\}$ be a multiset of ranking-based semantics. All semantics from $\mathscr{P}_1$ satisfy In. Consider $F_2 = (A,R)$ with its two connected components $F_2' = (\{a,b,c,d\},\{(a,b),(b,a),(a,d),(b,d),(d,b),(d,d),(d,c),(c,d),(c,c)\})$ and $F_2'' = (\{e,f\},\{(e,f)\}$ as depicted in Figure 2. The induced argument rankings are:*

$$\mathsf{Cat} : e \succ_{F_2}^{\mathsf{Cat}} a \succ_{F_2}^{\mathsf{Cat}} c \succ_{F_2}^{\mathsf{Cat}} b \succ_{F_2}^{\mathsf{Cat}} f \succ_{F_2}^{\mathsf{Cat}} d$$
$$\mathsf{Dbs} : e \succ_{F_2}^{\mathsf{Dbs}} a \succ_{F_2}^{\mathsf{Dbs}} f \succ_{F_2}^{\mathsf{Dbs}} c \succ_{F_2}^{\mathsf{Dbs}} b \succ_{F_2}^{\mathsf{Dbs}} d$$
$$\mathsf{2ZG} : e \succ_{F_2}^{\mathsf{2ZG}} a =_{F_2}^{\mathsf{2ZG}} b \succ_{F_2}^{\mathsf{2ZG}} f \succ_{F_2}^{\mathsf{2ZG}} c =_{F_2}^{\mathsf{2ZG}} d$$

The aggregated argument ranking of these rankings w.r.t to the Copeland$^\alpha$-based ranking-based semantics is $e \succ_{F_2}^{\mathsf{C}^\alpha(\mathscr{P}_1)} a \succ_{F_2}^{\mathsf{C}^\alpha(\mathscr{P}_1)} b =_{F_2}^{\mathsf{C}^\alpha(\mathscr{P}_1)} c =_{F_2}^{\mathsf{C}^\alpha(\mathscr{P}_1)} f \succ_{F_2}^{\mathsf{C}^\alpha(\mathscr{P}_1)} d$ and the corresponding ranking for F_2' is $a \succ_{F_2'}^{\mathsf{C}^\alpha(\mathscr{P}_1)} c \succ_{F_2'}^{\mathsf{C}^\alpha(\mathscr{P}_1)} b \succ_{F_2'}^{\mathsf{C}^\alpha(\mathscr{P}_1)} d$, for all $\alpha \in [0,1]$. Since $b =_{F_2}^{\mathsf{C}^\alpha(\mathscr{P}_1)} c$ while $c \succ_{F_2'}^{\mathsf{C}^\alpha(\mathscr{P}_1)} b$ for all $\alpha \in [0,1]$, In is violated by all Copeland$^\alpha$-based semantics, despite all ranking-based semantics in $\mathscr{P}_1$ satisfying this principle.

For the other two aggregated ranking-based semantics, In is also violated in this example.

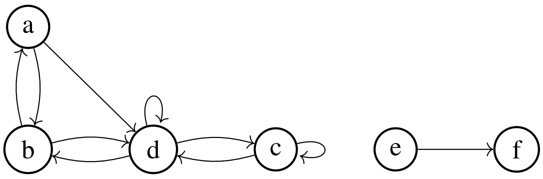

Figure 2. AF F_2 from Example 6.

Some principles are conceptually similar in a sense that they are independent from the ranking-based semantics.

Definition 15. *Let ρ be a ranking-based semantics and AF $F = (A,R)$. A principle P is called* $\succ$ / $=$ / $\succeq$-ranking-independent *if it consists of one implication, for which its antecedent is independent of the ranking-based semantics and the consequent is of the form $a \succ_F^\rho$ / $=_F^\rho$ / $\succeq_F^\rho$ b for arguments $a,b \in A$.*

	Cat	Dbs	2ZG	$\mathsf{pl}(\mathscr{P}_1)$	$\mathsf{B}^\alpha(\mathscr{P}_1)$	$\mathsf{C}^\alpha(\mathscr{P}_1)$
Abs	✓	✓	✓	✓	✓	✓
In	✓	✓	✓	✗	✗	✗
NaE	✓	✓	✓	✓	✓	✓
VP	✓	✓	✓	✓	✓	✓
SC	✗	✗	✓	✗	✗	✗
CP	✗	✓	✗	✗	✗	✗
QP	✗	✗	✗	✗	✗	✗
CT	✓	✓	✗	✗	✗	✗
SCT	✓	✓	✗	✗	✗	✗
AvsFD	✗	✗	✓	✗	✗	✗
σ-C*	✗	✗	✗	✗	✗	✗

Table 2. (non-)satisfied principles for $\alpha \in [0,1]$ and $\mathscr{P}_1 = \{\mathsf{Dbs}, \mathsf{Cat}, \mathsf{2ZG}\}$. * σ-C for $\sigma = \{co, pr, gr, stb\}$

Proposition 3. *NaE is $=$-ranking-independent and the principles VP, SC, CP, AvsFD, and σ-C are $\succ$-ranking-independent.*

The transferability of a ranking-independent principle for an aggregated ranking-based semantics is closely related to the ranking preservation of the aggregator used.

Proposition 4. *The following hold:*

- *If the aggregator Θ satisfies $\succ$-preservation, then $\succ$-ranking-independent principles are transferable for Θ.*
- *If the aggregator Θ satisfies $=$-preservation, then $=$-ranking-independent principles are transferable for Θ.*
- *If the aggregator Θ satisfies $\succeq$-preservation, then $\succeq$-ranking-independent principles are transferable for Θ.*
- *Abs is transferable by the aggregators pl, B^α and C^α.*

By combining Propositions 3 and 4 we can directly show that some principles are satisfied by aggregated ranking-based semantics derived from aggregators that satisfy preservation properties.

Proposition 5. *Let $\mathscr{P}$ be a multiset of ranking-based semantics. The following holds:*

- *If the aggregator Θ satisfies $\succ$-preservation, then the following principles are transferable by Θ: VP, SC, CP, AvsFD, σ-C.*
- *If the aggregator Θ satisfies $=$-preservation, then NaE is transferable by Θ.*

With Propositions 1 and 5 we can deduce that for our aggregators B^α and C^α principles VP, SC, CP, AvsFD, σ-C and NaE are transferable, while for pl only NaE is transferable.

Example 7. *Let $\mathscr{P}_1 = \{\mathsf{Dbs}, \mathsf{Cat}, \mathsf{2ZG}\}$. $\mathsf{pl}(\mathscr{P}_1)$, satisfies NaE, and $\mathsf{B}^\alpha(\mathscr{P}_1)$ and $\mathsf{C}^\alpha(\mathscr{P}_1)$ are satisfying NaE and VP for all $\alpha \in [0,1]$. The remaining satisfied and non-satisfied principles for this example are depicted in Table 2.*

5.2. Majority-transferability

For a small number of principles, multiple ranking-based semantics that satisfy this principle can be found. For instance, the three ranking-based semantics Dbs, Cat and 2ZG only share the principles Abs, In, NaE and VP. For all other principles at least one of the semantics violates it. Thus, considering only settings where all semantics satisfy a principle is too restrictive. Hence, we discuss a variation of transferability where only a majority of the ranking-based semantics need to satisfy a principle.

Definition 16. *A principle P is* majority-transferable *by an aggregator* Θ *if the following holds: If at least* $\lceil \frac{|\mathscr{P}|}{2} \rceil$ *semantics* $\rho \in \mathscr{P}$ *satisfy P, then also* $\Theta(\mathscr{P})$ *satisfies P for all multisets of ranking-based semantics* $\mathscr{P}$.

Note that if a principle is majority-transferable for aggregated ranking-based semantics, then it is also transferable for the aggregated ranking-based semantics.

Proposition 6. *If a principle P is majority-transferable by aggregator* Θ*, then P is transferable by* Θ*.*

The converse is not true. While a principle may be transferable by an aggregated ranking-based semantics, it might no longer be majority-transferable by allowing ranking-based semantics which violate the principle.

Some principles are still majority-transferable by our C^α aggregators.

Proposition 7. *VP, CP, SC and* σ*-C are majority-transferable by* C^α *with* $\alpha \in [0,1]$.

6. Conclusion and Future Work

In this paper, we investigated how to aggregate ranking-based semantics into one collective preorder. We have shown that the aggregated ranking based on the Borda rule and the Copeland rule preserves every relationship between arguments if all rankings induce the same relationship. Based on these results, we have demonstrated that the satisfaction of some principles can be transferred to the aggregated case, provided that all or at least the majority of the ranking-based semantics satisfy this principle. The principles Abs, NaE, VP, SC, CP, AvsFD, and σ-C are transferable for B^α and C^α, however VP, CP, SC and σ-C are only majority-transferable for C^α.

Several works have combined computational argumentation and voting. For example, Thomé et al. utilised voting rules to aggregate different attack relations and construct a collective AF [16]. Chen et al. extended this idea of aggregated AFs by analysing when semantic properties, such as conflict-free, are preserved in the aggregated AF [17]. Müller et al. used voting rules to eliminate cycles in an AF based on agents' preferences regarding arguments [18]. Bernreiter et al. enhanced an AF with a set of voters and their preferences over the arguments, then they utilised voting rules to select a σ-extension corresponding to the voters' preferences [19]. While all of these works use voting rules either to aggregate AFs or to select a σ-extension, none address ranking-based semantics.

In this paper, we focus on ranking-based semantics, which produce preorders. Thus, voting rules are the obvious first approach to aggregating them. However, as we have seen with the plurality voting rule, not all rules are appropriate. If we restrict ourselves to gradual semantics only, simpler aggregation approaches become suitable. Since gradual semantics return numerical strength values for each argument, we can use a weighted sum to aggregate these values of an argument among various gradual semantics. Therefore, it is interesting to investigate whether similar transferability results could be found for gradual semantics.

In the real world, the quality of an argument is multidimensional as shown by the argument mining community. Wachsmuth et al. discussed how to assess the quality of an argument into a single value [20,21]. In future work, we intend to adapt our approach to the natural language processing side of argumentation research.

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