Ranking-based Semantics for Assumption-based Argumentation

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Abstract
We present a general framework to rank assumptions in assumption-based argumentation frameworks (ABA frameworks), relying on their relationship to other assumptions and the syntactical structure of the ABA framework. We define general principles for assessing the suitability of ranking-based semantics and propose a new family of semantics for ABA frameworks that is using reductions to the abstract argumentation setting and leveraging existing ranking-based semantics for abstract argumentation. We show that this family complies with many of our principles.

Keywords
Argumentation, Structured Argumentation, Ranking

1. Introduction

In recent years, formal argumentation [1] has gained attention as a rational decision-making model. Formal argumentation is concerned with the representation of arguments and their relationships. One important approach is the abstract argumentation framework (AF) by Dung [2]. This framework uses directed graphs to represent arguments as nodes and attacks between two arguments as edges between these two arguments, where the source of an edge *attacks* the target. One way to reason with AFs is by using *extension-based semantics*, which are relying on functions allowing us to state when a set of arguments is jointly *acceptable*.

In addition to AFs, other models of rational decision-making using argumentative reasoning were defined in the literature. One of them are *assumption-based argumentation frameworks* (ABA frameworks) [3, 4, 5, 6]. These frameworks are based on deductive systems over a formal language and rules. One particular part of the formal language are the so-called *assumptions*, which are used as the basis for deriving further pieces of information. Similar to AFs, one reasoning method for ABA frameworks are extension-based semantics that state when a set of assumptions is *acceptable*. Abstract argumentation frameworks and ABA frameworks are closely related, the standard approach for reasoning with ABA frameworks includes the derivation of an AF and a translation for the other direction exists as well [7].

The classical semantics of both AFs and ABA frameworks induce a binary classification of arguments resp. assumptions: an argument or assumption is either accepted or not. This can be considered too limiting in real world scenarios like online debates [8]. For AFs, *ranking-based semantics* [9, 10] were introduced to overcome this limitation, where a ranking over the arguments based on their individual strength is established. Hence, we can not only state that an argument is part of an acceptable set or not, but infer that one argument is "better" than another one.

The ranking-based approach does not only allow us to establish whether one assumption is "better" than another one, but additionally we can use it to refine other reasoning methods. Let us assume we have two sets of assumptions S_1 and S_2 which are acceptable with respect to some extension-based semantics. Say these two sets are in a conflict with each other, i. e., we cannot accept both sets at the same time. With the help of the individual strength of each assumption, we can identify the "better" set of assumptions between them. For example S_1 might contain assumptions ranked higher than assumptions of S_2, hence we can consider S_1 to be better than S_2.

In this paper, we introduce *ranking-based semantics* for the ABA setting to rank assumptions based on their strength. With the help of these semantics we can state if an assumption is stronger than another one. To evaluate different ranking-based semantics approaches, we propose principles each describing different desirable behaviours for concrete approaches. With this principle-based approach we can compare different ranking-based semantics based solely on their behaviour. Additionally, we present a family of ranking-based semantics based on ideas for AFs. For an ABA framework, we look at the induced AF and calculate a ranking over arguments to then lift the resulting ranking back to ABA and then re-evaluate the result in the context of ABA.

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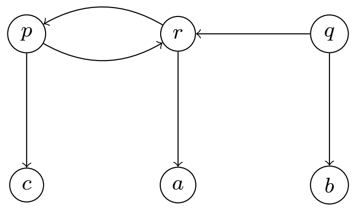

Figure 1: Abstract argumentation framework AF from Example 1.

This paper is organised as follows. We recall the necessary background information about AFs, ranking-based semantics and ABA frameworks in Section 2. In Section 3, we introduce ranking-based semantics for ABA frameworks, present principles for these semantics and propose a family of ranking-based semantics for ABA frameworks based on ranking-based semantics for AFs. Related work will be discussed in Section 4 and Section 5 will conclude the paper.

2. Preliminaries

Abstract Argumentation Frameworks

An *abstract argumentation framework* (AF) is a directed graph $F = (A, R)$ where A is a finite set of *arguments* and $R \subseteq A \times A$ is an *attack relation* [2]. An argument a is said to *attack* an argument b if $(a, b) \in R$. We say that an argument a is *defended by a set* $E \subseteq A$ if every argument $b \in A$ that attacks a is attacked by some $c \in E$. For $a \in A$ we define $a^-_F = \{b \mid (b, a) \in R\}$ and $a^+_F = \{b \mid (a, b) \in R\}$, so the sets of attackers of a and the set of arguments attacked by a in F. For a set of arguments $E \subseteq A$ we extend these definitions to E^-_F and E^+_F via $E^-_F = \bigcup_{a \in E} a^-_F$ and $E^+_F = \bigcup_{a \in E} a^+_F$, respectively. If the AF is clear in the context, we will omit the index.

Example 1. *Consider the argumentation framework $F = (A, R)$ depicted as a directed graph in Figure 1, with the nodes corresponding to arguments, and the edges corresponding to attacks*

$$A = \{a, b, c, p, q, r\}$$
$$R = \{(r, a), (q, b), (p, c), (p, r), (r, p), (q, r)\}.$$

Most semantics [11] for abstract argumentation are relying on two basic concepts: *conflict-freeness* and *admissibility.*

Definition 1 (Conflict-freeness, Admissibility)**.** *Given $F = (A, R)$, a set $E \subseteq A$ is*

- conflict-free *iff $\forall a, b \in E$, $(a, b) \notin R$;*
- admissible *iff it is conflict-free, and every element of E is defended by E.*

We use $cf(F)$ and $ad(F)$ for denoting the sets of conflict-free and admissible sets of an argumentation framework F, respectively. The intuition behind these concepts is that a set of arguments may be accepted only if it is internally consistent (conflict-freeness) and able to defend itself against potential threats (admissibility). The semantics proposed by Dung [2] are then defined as follows.

Definition 2 (Extension-based Semantics)**.** *Given $F = (A, R)$, an admissible set $E \subseteq A$ is*

- *a* complete *extension (co) iff it contains every argument that it defends;*
- *a* preferred *extension (pr) iff it is a $\subseteq$-maximal complete extension;*
- *the unique* grounded *extension (gr) iff it is the $\subseteq$-minimal complete extension;*
- *a* stable *extension (stb) iff $E^+_F = A \setminus E$.*

The sets of extensions of an argumentation framework F, for these four semantics, are denoted (respectively) $co(F)$, $pr(F)$, $gr(F)$ and $stb(F)$.

Ranking-based Semantics

Instead of only reasoning based on the acceptance of sets of arguments, *ranking-based semantics* [10] were introduced to focus on the strength of a single argument with respect to the other arguments. Note that the order returned by a ranking-based semantics is not necessarily total, i. e., not every pair of arguments is comparable.

Definition 3. *A* ranking-based semantics *ρ is a function, which maps an argumentation framework $F = (A, R)$ to a preorder*[1] *$\succeq^\rho_F$ on A.*

Intuitively, $a \succeq^\rho_F b$ means that a is at least as strong as b in F. We define the usual abbreviations as follows; $a \succ^\rho_F b$ denotes *strictly stronger*, i. e. $a \succeq^\rho_F b$ and $b \not\succeq^\rho_F a$. Moreover, $a \simeq^\rho_F b$ denotes *equally strong*, i. e. $a \succeq^\rho_F b$ and $b \succeq^\rho_F a$.

One example for ranking-based semantics is the *Burden-based semantics* [10]. This semantics calculates in each step a *burden number* for each argument based on the burden number of the attackers in the previous step and then lexicographically compares these numbers to establish a ranking.

Definition 4. *[10] Let $F = (A, R)$ be an AF. The* burden number *$bur(a)$ for argument $a \in A$ is denoted as $bur(a) = (bur_0(a), bur_1(a), bur_2(a), ...)$ whereby the*

[1] A preorder is a (binary) relation that is *reflexive* and *transitive*.

functions bur_i for $i \in \mathbb{N}$ are mapping arguments a to values as follows:

$$bur_i : A \to \mathbb{Q},$$

$$bur_i(a) := \begin{cases} 1 & \text{if } i = 0 \\ 1 + \sum_{b \in a_F^-} \frac{1}{bur_{i-1}(b)} & \text{otherwise} \end{cases}$$

The Burden-based semantics *(Bbs) defines a ranking $\succeq_F^{Bbs}$ s.t. for each $a, b \in A$, it holds that $a \succeq_F^{Bbs} b$ iff $bur(a) \succeq_{lex} bur(b)$, where $\succeq_{lex}$ is the* lexicographical order, *i. e. for two (possibly infinite) real number vectors $V = (V_1, V_2, \ldots)$ and $V' = (V_1', V_2', \ldots)$ we say $V \succ_{lex} V'$ iff $\exists i$ s.t. $V_i < V_i'$ and $\forall j < i$, $V_j = V_j'$ and we say $V \simeq_{lex} V'$ iff $\forall i$, $V_i = V_i'$.*

Example 2. *Given the AF F from Example 1. We calculate for each argument the burden number. Argument q is unattacked, hence $bur(q) = (1, 1, 1, \ldots)$. Based on the value of q, we can calculate the remaining burden numbers:*

$$\begin{aligned} bur(a) &= (1, 2, \frac{4}{3}, \ldots); \\ bur(b) &= (1, 2, 2, \ldots); \\ bur(c) &= (1, 2, \frac{2}{3}, \ldots); \\ bur(p) &= (1, 2, \frac{4}{3}, \ldots); \\ bur(r) &= (1, 3, 2.5, \ldots). \end{aligned}$$

Since a and p have the same attacker r, they receive in each step the same value. So, these burden numbers result in the following ranking:

$$q \succ_F^{Bbs} a \simeq_F^{Bbs} p \succ_F^{Bbs} c \succ_F^{Bbs} b \succ_F^{Bbs} r.$$

Argument q is ranked highest, then a and p are equally strong, then c followed by b, and finally the least ranked argument is r.

We recall some of the most fundamental principles [9] that guide the development of ranking-based semantics for abstract argumentation. The first and most basic principle states, that the names of the arguments should not influence the ranking.

Definition 5 (Isomorphism). *An* isomorphism *γ between two argumentation frameworks $F = (A, R)$ and $F' = (A', R')$ is a bijective function $\gamma : A \to A'$ such that for all $a, b \in A$, $(a, b) \in R$ iff $(\gamma(a), \gamma(b)) \in R'$.*

Definition 6 (Abs). *A ranking-based semantics ρ satisfies* Abstraction *(short Abs) if for every pair of AFs $F = (A, R)$, $F' = (A', R')$ and every isomorphism $\gamma : A \to A'$, for all $a, b \in A$, we have $a \succeq_F^\rho b$ iff $\gamma(a) \succeq_{F'}^\rho \gamma(b)$.*

The next principle states, that unattacked arguments should be ranked better than any attacked argument.

Definition 7 (VP). *A ranking-based semantics ρ satisfies* Void Precedence *(short VP) iff for any AF $F = (A, R)$, it holds that for all $a, b \in A$ with $a_F^- = \emptyset$ and $b_F^- \neq \emptyset$, $a \succ_F^\rho b$.*

Contrasting to VP the principle SC focuses on the worst ranked arguments. These worst arguments are self-conflicting arguments and any self-conflicting argument should be ranked worse than any not self-conflicting argument.

Definition 8 (SC). *A ranking-based semantics ρ satisfies* Self-Contradiction *(short SC) iff for any AF $F = (A, R)$, it holds that for all $a, b \in A$ with $(a, a) \notin R$ and $(b, b) \in R$, $a \succ_F^\rho b$.*

The principle *Cardinality Precedence* focuses on the number of attackers. If an argument has fewer attackers than another argument, the first argument can be considered stronger.

Definition 9 (CP). *A ranking-based semantics ρ satisfies* Cardinality Precedence *(short CP) iff for any AF $F = (A, R)$, it holds that for all $a, b \in A$ with $|a^-| < |b^-|$, $a \succ_F^\rho b$.*

The final principle states that any of two arguments should be comparable.

Definition 10 (Total). *A ranking-based semantics ρ satisfies* Total *iff for any AF $F = (A, R)$, it holds that for all $a, b \in A$ either $a \succeq_F^\rho b$ or $b \succeq_F^\rho a$.*

Note that, this is not a complete list of principles, more principles can be found in the literature [9] and also these principles are not mandatory principles since there are incompatibilities between them, e.g., CP and SC are incompatible. *Bbs* satisfies Abs, VP, CP and Total and violates SC [9].

Assumption-based Argumentation Frameworks

Assumption-based Argumentation (ABA) uses a deductive system $(\mathcal{L}, \mathcal{R})$, where $\mathcal{L}$ is a formal language and $\mathcal{R}$ a set of rules of the form $r = a_0 \leftarrow a_1, \ldots, a_n$ with $a_i \in \mathcal{L}$. We say that a_0 is the head of the rule ($head(r) = a_0$) and the set $\{a_1, \ldots, a_n\}$ is the body ($body(r) = \{a_1, \ldots, a_n\}$).

Definition 11. *An* ABA framework *is a tuple $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$, where $(\mathcal{L}, \mathcal{R})$ is a deductive system, $\mathcal{A} \subseteq \mathcal{L}$ a non-empty set of assumptions, and $\overline{} : \mathcal{A} \to \mathcal{L}$ is a so-called contrary function.*

We focus in this work on *flat* ABA frameworks, i. e., $head(r) \notin \mathcal{A}$ for each rule $r \in \mathcal{R}$.

A sentence $s \in \mathcal{L}$ is derivable from a set of assumptions $X \subseteq \mathcal{A}$ and rules $R \subseteq \mathcal{R}$, denoted by $X \vdash_R s$, if there is a finite rooted labelled tree T with the root being labelled with s, the set of labels for the leaves of T is equal to X or $X \cup \{\top\}$, and the internal nodes are labelled with $head(r)$ according to a rule $r \in R$ s.t. the children are labelled with $body(r)$ or $\top$ if the body is empty. Each assumption $x \in X$ has an associated leaf labelled with x and each rule $r \in R$ has an associated node in the tree. For a tree T, we denote by $asm(T)$ the set of assumptions used to derive the conclusion denoted $cl(T)$ with rules $ru(T)$.

Similar to AFs, ABA frameworks can be used as a rational decision-making model. In order to reason with ABA frameworks, *extension-based semantics* were introduced to state when a set of assumptions is acceptable. A set of assumptions S *attacks* a set of assumptions $Q \subseteq \mathcal{A}$ if there is $S' \subseteq S, R \subseteq \mathcal{R}$, s.t. $S' \vdash_R \overline{a}$ for some $a \in Q$. S is *conflict-free* if S does not attack S. S *defends* assumption s if S attacks each assumption set Q that attacks $\{s\}$.

Definition 12. *For $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \text{-})$ be an ABA framework and a conflict-free set of assumptions $S \subseteq \mathcal{A}$, we say S is*

- admissible *in D ($S \in ad(D)$) if S defends itself,*
- complete *in D ($S \in co(D)$) if S is admissible and contains every assumptions set it defends,*
- grounded *in D ($S \in gr(D)$) if S is $\subseteq$-minimally complete,*
- preferred *in D ($S \in pr(D)$) if S is $\subseteq$-maximally complete, and*
- stable *in D ($S \in st(D)$) iff S attacks every assumption $a \in \mathcal{A} \setminus S$.*

Example 3. *Consider the ABA framework D with assumptions $\mathcal{A} = \{a, b, c\}$ and rules:*

$$r_1 : r \leftarrow b, c;$$
$$r_2 : q \leftarrow \ ;$$
$$r_3 : p \leftarrow q, a$$

with $\overline{a} = r, \overline{b} = q, \overline{c} = p$. Then for example, we can derive p from $\{a\}$ with rules r_2 and r_3 and since $p = \overline{c}$ we see that $\{a\}$ attacks $\{c\}$. Additionally, we see that $\{a\}$ and $\emptyset$ are the two admissible sets.

AFs and ABA frameworks are closely related [7], and we can define an AF as an instance of an ABA framework and the other way around.

Definition 13. *The associated AF $F_D = (A, R)$ of an ABA framework $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{\ })$ is given by $A = \{T \mid T$ is a tree for $s \in \mathcal{L}$ with $cl(T) = s\}$ and attack relation $(T, T') \in R$ iff there is $c \in asm(T')$ s.t. $\overline{c} = cl(T)$.*

Definition 14. *Let $F = (A, R)$ be an AF. The associated ABA framework of F is $ABA(F) = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{\ })$ with*

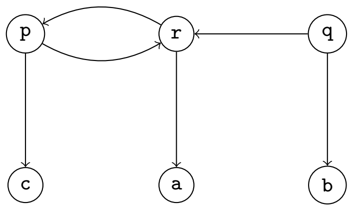

Figure 2: Graph representation of Example 4

- $\mathcal{A} = A$,
- $\mathcal{L} = \mathcal{A} \cup \{a^c | a \in \mathcal{A}\}$,
- $\mathcal{R} = \{b^c \leftarrow a | (a, b) \in R\}$,
- *for all $a \in \mathcal{A}$: $\overline{a} = a^c$.*

Cyras and Toni [7] showed that the acceptance coincides. So, if a set of assumptions S is acceptable in the ABA framework D, then S is also acceptable in the corresponding AF F_D (in the form of conclusions of an extension).

Example 4. *Continuing Example 3, we can construct the corresponding AF $F_D = (A, R)$ of D, with $A = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{p}, \mathtt{q}, \mathtt{r}\}$ where*

- $\mathtt{a}$ *is a tree with $asm(\mathtt{a}) = \{a\}$, $cl(\mathtt{a}) = a$, and $ru(\mathtt{a}) = \emptyset$,*
- $\mathtt{b}$ *is a tree with $asm(\mathtt{b}) = \{b\}$, $cl(\mathtt{b}) = b$, and $ru(\mathtt{b}) = \emptyset$,*
- $\mathtt{c}$ *is a tree with $asm(\mathtt{c}) = \{c\}$, $cl(\mathtt{c}) = c$, and $ru(\mathtt{c}) = \emptyset$,*
- $\mathtt{p}$ *is a tree with $asm(\mathtt{p}) = \{a\}$, $cl(\mathtt{p}) = p$, and $ru(\mathtt{p}) = \{r_3\}$,*
- $\mathtt{q}$ *is a tree with $asm(\mathtt{q}) = \emptyset$, $cl(\mathtt{q}) = q$, and $ru(\mathtt{q}) = \{r_2\}$,*
- $\mathtt{r}$ *is a tree with $asm(\mathtt{r}) = \{b, c\}$, $cl(\mathtt{r}) = r$, and $ru(\mathtt{r}) = \{r_1\}$*

and the attack relation

$$R = \{(\mathtt{q}, \mathtt{b}), (\mathtt{q}, \mathtt{r}), (\mathtt{r}, \mathtt{a}), (\mathtt{r}, \mathtt{p}), (\mathtt{p}, \mathtt{r}), (\mathtt{p}, \mathtt{c})\}.$$

The corresponding graph representation can be found in Figure 2. So, for each derivable sentence in an ABA framework, we create an argument in the corresponding AF. We know that p is derivable from $\{a\}$ by rules r_2 and r_3, hence $\mathtt{p} \in A$ and additionally the attacks in the AF are representing the attacks from one set of assumptions to another set of assumptions. For example, the attack $(\mathtt{p}, \mathtt{r}) \in R$ is representing the fact, that $\{a\}$ attacks $\{b\}$.

Note that in the following, we call argument $\mathtt{a}$, based on a tree of the form $asm(\mathtt{a}) = \{a\}$, $cl(\mathtt{a}) = a$ and $ru(\mathtt{a}) = \emptyset$, where a is an assumption, the *assumption argument* of a.

3. Ranking Assumptions

Up to this point, reasoning in ABA is focused on sets of assumptions being acceptable with respect to a semantics like admissible, complete, or preferred semantics. So, we can state that an assumption is contained in an acceptable set of assumptions or not, however this reasoning approach does not give us any insight into the strength of individual assumptions. In this paper, we shift the focus to the strength of the individual assumptions. For that purpose, we develop a general framework for ranking assumptions in ABA. This framework allows us to state that an assumption is stronger than another one.

Definition 15. *A ranking-based semantics τ is a function that maps an ABA framework $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$ to a preorder $\succeq^\tau_D$ on $\mathcal{A}$.*

Intuitively, $a \succeq^\tau_D b$ means, that assumption a is at least as strong as b in D. We define the usual abbreviations as follows;

- $a \succ^\tau_D b$ denotes *strictly stronger*, i.e. $a \succeq^\tau_D b$ and $b \not\succeq^\tau_D a$.
- $a \simeq^\tau_D b$ denotes *equally strong*, i.e. $a \succeq^\tau_D b$ and $b \succeq^\tau_D a$.

3.1. Principles

In order to evaluate different ranking-based approaches for ABA frameworks, we will follow a principle-based approach, like typical in the area of argumentation [12, 9]. These principles will give us insight into the behaviour of the different ranking-based semantics, hence allowing us to compare these different approaches. Note that all these principles are not mandatory and should be selected depending on the application. Some application favour some principles over others. In some other applications, we want to avoid satisfying particular principles, since these principles are obstructing or counter-intuitive in these scenarios. Next, we will propose a number of principles for ranking-based semantics for ABA frameworks.

The first principle states that an assumption for which we cannot derive the contrary should be ranked higher than any assumption for which we can derive the contrary.

Definition 16 (P1)**.** *Ranking-based semantics τ satisfies* P1 *iff for every ABA framework $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$ it holds that for every assumption $a \in \mathcal{A}$ s.t. $\overline{a}$ is not derivable from any set of assumptions $Q \subseteq \mathcal{A}$ and for every assumption $b \in \mathcal{A}$ s.t. $\overline{b}$ is derivable it holds that $a \succ^\tau_D b$.*

In other words, if there is no reason to lower the strength of an assumption, then the strength of that assumption should not be lowered.

One simple principle states that the names of the assumptions do not influence the ranking.

Definition 17 (Isomorphism)**.** *An* isomorphism *γ between two ABA frameworks $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$ and $D' = (\mathcal{L}', \mathcal{R}', \mathcal{A}', \overline{}')$ is a bijective function $\gamma : \mathcal{A} \to \mathcal{A}'$ (extended to $\mathcal{L}'$ with for all $x \in \mathcal{L} \setminus \mathcal{A}$: $\gamma(x) = x$ and $\mathcal{R}' = \{\gamma(r) | r \in \mathcal{R}\}$, where for all $r \in \mathcal{R}$ with $r = h \leftarrow a_1, ..., a_n$: $\gamma(h \leftarrow a_1, ..., a_n) = \gamma(h) \leftarrow \gamma(a_1), ..., \gamma(a_n))$ and $\overline{a} = \overline{\gamma(a)}'$ for all $a \in \mathcal{A}$.*

Definition 18 (P2)**.** *Ranking-based semantics τ satisfies* P2 *iff for every pair of ABA frameworks $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \text{-})$ and $D' = (\mathcal{L}', \mathcal{R}', \mathcal{A}', \text{-}')$ and for all isomorphisms γ s.t. $D' = \gamma(D)$, for all $a, b \in \mathcal{A}$, we have $a \succeq^\tau_D b$ iff $\gamma(a) \succeq^\tau_{D'} \gamma(b)$.*

The next principle is focusing on the addition of rules, where the head is a contrary of an assumption. In a sense, these rules can be considered as attacking rules. The addition of such a rule for an assumption should not raise the strength of that assumption.

Definition 19 (P3)**.** *Let $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$ be an ABA framework and $a \in \mathcal{A}$. Let r^-_{add} be a rule with $r^-_{add} \notin \mathcal{R}$ and $head(r^-_{add}) = \overline{a}$. D^-_{add} is a copy of D with r^-_{add} added, i. e., $D^-_{add} = (\mathcal{L}, \mathcal{R} \cup \{r^-_{add}\}, \mathcal{A}, \overline{})$.*

Ranking-based semantics τ satisfies P3 *iff for all ABA frameworks $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$ it holds for all $a, b \in \mathcal{A}$ with $a \neq b$ that $a \succeq^\tau_{D^-_{add}} b$ implies $a \succeq^\tau_D b$.*

So, the addition of rules, which in a sense can lower the strength of an assumption, should at least not raise the strength of that assumption.

Cyras and Toni [7] have shown that the acceptance of extension-based semantics coincides for ABA frameworks and their corresponding AFs. However, the transformation from an ABA framework to an AF and back to an ABA framework does add new rules and therefore changes the framework. The next principle ensures that these transformations between frameworks do not change the resulting ranking.

Definition 20 (P4)**.** *Ranking-based semantics τ satisfies* P4 *iff for every ABA framework $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$ and F_D the corresponding AF to D, and $ABA(F_D)$ the corresponding ABA framework to F_D, it holds for any pair $a, b \in \mathcal{A}$ that we have $a \succeq^\tau_D b$ iff $a \succeq^\tau_{ABA(F_D)} b$.*

The next principle focuses on assumptions, for which we can derive the contrary by only using the assumption itself. These assumptions are in a sense self-attacking and should be ranked worse than any non-self-attacking assumption.

Definition 21 (P5)**.** *Ranking-based semantics τ satisfies* P5 *iff for every ABA framework $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$ the following holds for every assumptions $a, b \in \mathcal{A}$, if $\{a\} \nvdash_\mathcal{R} \overline{a}$ and $\{b\} \vdash_\mathcal{R} \overline{b}$ then $a \succ^\tau_D b$.*

3.2. Methods

We define a family of ranking-based semantics for ABA frameworks that relies on the reduction of an ABA framework to its corresponding AF, an application of a ranking-based semantics for AFs on this derived AF, and a reinterpretation of the resulting ranking over arguments in terms of assumptions.

Definition 22. *Let $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$ be an ABA framework, $F_D = (A, R)$ the corresponding AF, $a, b \in \mathcal{A}$, $\mathtt{a}$, $\mathtt{b}$ the corresponding assumptions arguments, and ρ a ranking-based semantics for AFs. The ranking-based semantics ABA-ρ returns $a \succeq_D^{ABA\text{-}\rho} b$ iff $\mathtt{a} \succeq_{F_D}^{\rho} \mathtt{b}$.*

In other words, assumption a is at least as strong b in D if the corresponding assumption argument $\mathtt{a}$ is at least as strong as $\mathtt{b}$ in the corresponding AF of D.

Example 5. *In the following example, we use the Burden-based semantics as an example ranking-based semantics. Other semantics can be applied equivalently. Consider the ABA framework D from Example 3 and its corresponding AF F_D constructed in Example 4.*

Similar to Example 2, if we apply the Burden-based semantics to F_D, the resulting ranking is:

$$\mathtt{q} \succ_{F_D}^{Bbs} \mathtt{p} \simeq_{F_D}^{Bbs} \mathtt{a} \succ_{F_D}^{Bbs} \mathtt{c} \succ_{F_D}^{Bbs} \mathtt{b} \succ_{F_D}^{Bbs} \mathtt{r}.$$

Restricting the ranking to only assumption arguments gives us

$$\mathtt{a} \succ_{F_D}^{Bbs} \mathtt{c} \succ_{F_D}^{Bbs} \mathtt{b}.$$

We can project this ranking back to ABA:

$$a \succ_D^{ABA\text{-}Bbs} c \succ_D^{ABA\text{-}Bbs} b$$

Hence, a is the strongest assumption, then c, and b is the weakest assumption. The preferred extension of D is $\{a\}$, thus it is intuitive that a is the strongest assumption. While b is attacked by a fact $q \leftarrow$ meaning that b is not really strong and therefore should be ranked below c.

So, the corresponding AF of an ABA framework gives us insight into the relationship between each assumption. We see that if the corresponding argument is strong or highly ranked in the corresponding AF, then the assumption will also be strong in the ABA framework as well. Additionally, we can compare two assumptions b and c with each other, which is not possible by using solely extension-based semantics since both assumptions are not acceptable.

In the remainder of this section, we look at the compliance of this family of ranking-based semantics with respect to our principles from before. For that we look at the principles the underlying ranking-based semantics for AFs satisfies. With the help of these principles, we can show that principles in the ABA setting are satisfied.

The idea that unattacked arguments should be ranked best can be translated into the ABA setting. The unattacked arguments then correspond to assumptions for which we can not derive the contrary.

Theorem 1. *If ρ satisfies VP, then ABA-ρ satisfies* P1.

Proof. Let $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$ be an ABA framework, $F_D = (A, R)$ the corresponding AF, $a, b \in \mathcal{A}$, $\mathtt{a}$, $\mathtt{b}$ the corresponding assumptions arguments, and ρ a ranking-based semantics for AFs.

Assume ρ satisfies VP, $\overline{a}$ is not derivable and $\overline{b}$ is derivable. Since $\overline{a}$ is not derivable, we know that $\mathtt{a}$ can not be attacked in F_D, because we do not have any argument $\mathtt{x}$ in F_D with $cl(\mathtt{x}) = \overline{a}$. Hence, $\mathtt{a}_{F_D}^- = \emptyset$. Additionally, we know that $\mathtt{b}$ is attacked at least once, because $\overline{b}$ is derivable in D, so there has to be an argument $\mathtt{x}'$ s.t. $cl(\mathtt{x}') = \overline{b}$. Hence, $\mathtt{b}_{F_D}^- \neq \emptyset$. Since ρ satisfies VP, we know that $\mathtt{a} \succ_{F_D}^{\rho} \mathtt{b}$ and therefore also $a \succ_D^{\text{ABA-}\rho} b$. $\square$

We see that the names of assumptions do not influence the ranking, despite translating the ABA framework into an AF.

Theorem 2. *If ρ satisfies Abs, then ABA-ρ satisfies* P2.

Proof. Let $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$ and $D' = (\mathcal{L}', \mathcal{R}', \mathcal{A}', \overline{}')$ be two ABA frameworks and γ be an isomorphism s.t. $D' = \gamma(D)$. Let $F_D = (A, R)$ resp. $F_{D'} = (A', R')$ be the corresponding AFs for D and D'. Let ρ be an ranking-based semantics for AFs.

Assume ρ satisfies Abs. We know that for every assumption $a \in \mathcal{A}$ there is an isomorphic assumption $a' \in \mathcal{A}'$, hence for every argument $\mathtt{a} \in A$ there is an isomorphic assumption argument $\mathtt{a}'$ in $F_{D'}$. Similar can be reason for any other element in D. Therefore, there has to be an isomorphism γ' for F_D s.t. $\gamma'(F_D) = F_{D'}$. Since ρ satisfies Abs, we know that for any pair of arguments $(\mathtt{a}, \mathtt{b})$ it holds that if $\mathtt{a} \succeq_{F_D}^{\rho} \mathtt{b}$ then $\gamma'(\mathtt{a}) \succeq_{F_{D'}}^{\rho} \gamma'(\mathtt{b})$. Hence, it holds that $\gamma(a) \succeq_{D'}^{\text{ABA-}\rho} \gamma(b)$ iff $a \succeq_D^{\text{ABA-}\rho} b$ and therefore *P2* is satisfied. $\square$

In the following, we show that if the underlying ranking-based semantics for AFs satisfies CP and Total, then we know that the addition of rules, with which we can derive the contrary of an assumption, do not raise the strength of that assumption.

Theorem 3. *If ρ satisfies CP and Total, then ABA-ρ satisfies* P3.

Proof. Let $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$ be an flat ABA framework, $F_D = (A, R)$ the corresponding AF and ρ a ranking-based semantics for AFs. Let r_{add}^- is a new rule for $a \in \mathcal{A}$, where $r_{add}^- \notin \mathcal{R}$ and $head(r_{add}^-) = \overline{a}$ and D_{add}^- is a copy of D with r_{add}^- added, i.e. $D_{add}^- = (\mathcal{L}, \mathcal{R} \cup \{r_{add}^-\}, \mathcal{A}, \overline{})$ and let $F_{D_{add}^-}$ be the corresponding AF.

Assume ρ satisfies CP, Total and $a \succeq^{\text{ABA-}\rho}_{D^-_{add}} b$ for $b \in \mathcal{A}$ and the corresponding assumption arguments $\mathtt{a}$ and $\mathtt{b}$. First, we look at the case that r^-_{add} can not be activated, so there is no tree $\mathtt{x}$ s.t. $r^-_{add} \in ru(\mathtt{x})$ meaning that, $body(r^-_{add}) \nsubseteq \mathcal{A}$ and there is no sequence of rules $(r_1, ..., r_n, r^-_{add})$ from $\mathcal{R}$ s.t. $body(r^-_{add}) \subseteq \bigcup_{i=1}^{n} head(r_i) \cup \mathcal{A}$. Then the addition of r^-_{add} does not change the corresponding AF, i.e. $F_D = F_{D^-_{add}}$ and therefore $a \succeq^{\text{ABA-}\rho}_{F_{D^-_{add}}} b$ implies $a \succeq^{\text{ABA-}\rho}_{F_D} b$.

Next, we look at the case, where r^-_{add} can be activated. The addition of any attack into an AF can only raise the number of attackers for an argument and can not lower the number of attackers. Similar hold for ABA frameworks, the addition and activation of a new rule does not yield to deactivation of other rules. Hence, it holds that $|\mathtt{x}^-_{F_D}| \leq |\mathtt{x}^-_{F_{D^-_{add}}}|$ for any $x \in \mathcal{A}$ and its corresponding assumption argument $\mathtt{x}$. Since ρ satisfies CP and Total and $a \succeq^{\rho}_{D^-_{add}} b$ holds, we know that $|\mathtt{a}^-_{F_{D^-_{add}}}| \leq |\mathtt{b}^-_{F_{D^-_{add}}}|$.

If $|\mathtt{b}^-_{F_D}| = |\mathtt{b}^-_{F_{D^-_{add}}}|$, then it is clear that $|\mathtt{a}^-_{F_D}| \leq |\mathtt{b}^-_{F_D}|$ and since ρ satisfies CP and Total it holds that $\mathtt{a} \succeq^{\rho}_{F_D} \mathtt{b}$ and therefore also $a \succeq^{\text{ABA-}\rho}_{D} b$.

For $|\mathtt{b}^-_{F_D}| < |\mathtt{b}^-_{F_{D^-_{add}}}|$ we know that we can derive $\overline{a}$ in $F_{D^-_{add}}$ and this activates a rule r' with $\overline{a} \in body(r')$ and this rule is needed to activate rule r'' with $head(r'') = \overline{b}$. This implies that $\overline{a}$ can not be derived in D otherwise we could activate r' in D as well and that means that $|\mathtt{b}^-_{F_D}| < |\mathtt{b}^-_{F_{D^-_{add}}}|$ could not hold. Since $\overline{a}$ can not be derived this implies $|\mathtt{a}^-_{F_D}| = 0$ and therefore $|\mathtt{a}^-_{F_D}| \leq |\mathtt{b}^-_{F_D}|$ and also $\mathtt{a} \succeq^{\rho}_{F_D} \mathtt{b}$, which implies $a \succeq^{\text{ABA-}\rho}_{D} b$. $\square$

Similar to extension-based reasoning transforming ABA frameworks into AF and back should not influence the resulting ranking.

Theorem 4. *If ρ satisfies CP and Total, then ABA-ρ satisfies* P4.

Proof. Let $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, ^-)$ be a flat ABA framework, $F_D = (A, R)$ the corresponding AF, $ABA(F_D)$ the corresponding ABA framework of F_D, $F_{ABA(F_D)}$ the corresponding AF to $ABA(F_D)$ and ρ a ranking-based semantics for AFs. Let $a, b \in A$, $\mathtt{a}$ be the corresponding assumptions argument of a and $\mathtt{b}$ be the corresponding assumption argument of b.

Assume ρ satisfies CP, Total and $a \succeq^{\text{ABA-}\rho}_{D} b$. If a sentence is derivable in D, then there is a corresponding argument in F_D and every argument in F_D is an assumption in $ABA(F_D)$ and since assumptions are always derivable, we know that everything, which is derivable in D is also derivable in $ABA(F_D)$. This implies that the number of attacker for any assumption argument $\mathtt{a}$ in F_D is equal to the number of attacker for the corresponding assumption argument in $F_{ABA(F_D)}$. Since ρ satisfies CP and Total and $a \succeq^{ABA-\rho}_{D} b$, we know $|\mathtt{a}^-_{F_D}| \leq |\mathtt{b}^-_{F_D}|$ and since the number of attacker is the same in F_D and $F_{ABA(F_D)}$, i.e. $|(a)^-_{F_D}| = |(a)^-_{F_{ABA(F_D)}}|$, we have $|\mathtt{a}^-_{F_{ABA(F_D)}}| \leq |\mathtt{b}^-_{F_{ABA(F_D)}}|$. CP and Total then imply $\mathtt{a} \succeq^{\rho}_{F_{ABA(F_D)}} b$ and therefore also $a \succeq^{\text{ABA-}\rho}_{ABA(F_D)} b$. $\square$

If the underlying ranking-based semantics ρ does satisfy SC, then we know that in the ABA setting assumptions for which we can derive the contrary by only using the assumption should be ranked worst.

Theorem 5. *If ρ satisfies SC, then ABA-ρ satisfies* P5.

Proof. Let $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, ^-)$ be an ABA framework, $F_D = (A, R)$ the corresponding AF, $a, b \in \mathcal{A}$, the corresponding assumptions arguments $\mathtt{a}$, $\mathtt{b}$, and ρ a ranking-based semantics for AFs.

Assume ρ satisfies SC and $\{a\} \nvdash_{\mathcal{R}} \overline{a}$ and b with $\{b\} \vdash_{\mathcal{R}} \overline{b}$. This implies that $(\mathtt{b}, \mathtt{b}) \in R$ and $(\mathtt{a}, \mathtt{a}) \notin R$. So, $\mathtt{b}$ attacks itself and also an assumption argument $\mathtt{x}$ for $x \in \mathcal{A}$ can only attack it self if $\{x\} \vdash_{\mathcal{R}} \overline{x}$, hence $\mathtt{a}$ can not attack it self. Because ρ satisfies SC, we know $\mathtt{a} \succ^{\rho}_{F_D} \mathtt{b}$ and this implies $a \succ^{\text{ABA-}\rho}_{D} b$. $\square$

The principles the underlying ranking-based semantics for AFs need to satisfy in order to satisfy every principle proposed for the ABA setting are simple. A number of different ranking-based semantics for AFs are suitable to be used, since they satisfy a good number of principles like the Burden-based semantics satisfies Abs, VP, CP and Total and therefore ABA-Bbs satisfies *P1, P2, P3* and *P4.* However, Besnard et al. [13] have shown that a few principles for the AF setting are incompatible with each other, in particular CP and SC are incompatible. Hence, there is no ranking-based semantics, which satisfies CP and SC. Therefore, we have to check the principles in the ABA setting by hand. For ABA-Bbs we know, that SC is violated, therefore we have to check *P5.* By adapting the counterexample used to show the incompatibility of CP and SC, we can show that ABA-Bbs does violate *P5* [13, 14].

Example 6. *Let $D_1 = (\mathcal{L}, \mathcal{R}, \mathcal{A}, ^-)$, with $\mathcal{A} = \{a, b, c\}$, rules $r_1 : b \to \overline{a}$, $r_2 : c \to \overline{a}$, and $b = \overline{b}$. The corresponding AF is*

$$F_{D_1} = (\{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{r}_1, \mathtt{r}_2\}, \{(\mathtt{r}_1, \mathtt{a}), (\mathtt{r}_2, \mathtt{a}), (\mathtt{b}, \mathtt{b})\}),$$

where

- $\mathtt{a}$ *is a tree with* $asm(\mathtt{a}) = \{a\}$, $cl(\mathtt{a}) = a$ *and* $ru(a) = \emptyset$,

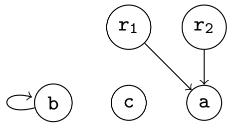

Figure 3: Graph representation of Example 6

- $\mathtt{b}$ *is a tree with* $asm(\mathtt{b}) = \{b\}$*,* $cl(\mathtt{b}) = b$ *and* $ru(b) = \emptyset$*,*
- $\mathtt{c}$ *is a tree with* $asm(\mathtt{c}) = \{c\}$*,* $cl(\mathtt{c}) = c$ *and* $ru(c) = \emptyset$*,*
- $\mathtt{r}_1$ *is a tree with* $asm(\mathtt{r}_1) = \{b\}$*,* $cl(\mathtt{r}_1) = \overline{a}$ *and* $ru((r_1) = \{r_1\}$*,*
- $\mathtt{r}_2$ *is a tree with* $asm(\mathtt{r}_2) = \{c\}$*,* $cl(\mathtt{r}_2) = \overline{a}$ *and* $ru((r_2) = \{r_2\}$*.*

Depicted in Figure 3.

So, b *implies its contrary* $\{b\} \vdash_{\emptyset} \overline{b}$ *and* a *does not imply its contrary. When we apply the Burden-based semantics, we see that* $bur(\mathtt{a}) = (1, 3, 3, ...)$ *and* $bur(\mathtt{b}) = (1, 2, 2, ...)$*, this implies* $\mathtt{b} \succ^{Bbs}_{F_{D_1}} \mathtt{a}$ *and therefore* $b \succ^{ABA\text{-}Bbs}_{D_1} a$*. However, this contradicts* $a \succ^{ABA\text{-}Bbs}_{D_1} b$ *like* P5 *implies.*

4. Related Work

One of the most discussed topics in structured argumentation are preferences over uncertain information. These preferences state that information a is better or more believable than information b. A number of frameworks working with preferences can be found the literature like ASPIC$^+$ [15, 16, 17, 18, 19], ABA$^+$ [7, 20] or p_ABA [21]. While ABA$^+$ and p_ABA are extensions of ABA, ASPIC$^+$ is a general-purpose structure argumentation framework, with focus on preferences. Prakken [17] has shown that flat ABA frameworks can be instantiated as ASPIC$^+$ frameworks. ABA$^+$ receives in addition of an ABA framework a preference over the assumptions as an input. Using these preferences a new attack relation is defined. Similar to ABA$^+$, p_ABA receives in addition to the ABA framework a preference as an input. However, the preference in p_ABA is over the sentences $\mathcal{L}$.

In these frameworks the preferences are preorders over rules and ordinary premises (ASPIC$^+$), assumptions (ABA$^+$) or sentences (p_ABA). Hence, these preferences are similar to our rankings over assumptions. All these preferences can be seen as a strength notion, if an assumption a is preferred over assumption b in an ABA$^+$ framework, then this relationship between a and b can be seen as a is better than b. However, all these frameworks receive their preferences as an input rather than calculating the preorder based on the language and rules given.

In ASPIC$^+$ and ABA$^+$ preferences are used to disable or reverse attacks. If the target of an attack is considered better than the attacker, this attack is discarded or reversed, so the attacker is the attacked.

One interesting idea with ABA$^+$ is to use the underlying ranking over assumptions to construct the corresponding ABA$^+$ framework for an ABA framework. So, we take an ABA framework and calculate a ranking over the assumption with any ranking-based semantics like ABA-Bbs to then construct an ABA$^+$ framework using our ranking as a preference order. An ABA$^+$ framework constructed in such a way has similarities with the underlying ABA framework for example the conflict-free sets are the same. Hence, we can transform any ABA framework into an ABA$^+$ framework without additional information like a preference order.

p_ABA uses preferences to discredit sets of assumptions. Wakaki [21] proposes preorders over sets of assumptions. However, their approach has two big differences: first, in p_ABA preferences are part of the input and, secondly, they can only differentiate sets of assumptions, which belong to an extension-based semantics.

In the literature, ranking-based semantics are used to refine extension-based reasoning in the area of abstract argumentation. For example Bonzon et al. [22] uses the aggregated strength values of each argument of a set to compare two sets. While Konieczny, Marquis, and Vesic [23] are comparing two sets of arguments using a pairwise comparison based on a criterion like the number of arguments inside the first set not attacked by the second set. So, the presented ranking-based semantics for ABA frameworks are the first step towards refining extension-based reasoning inside structured argumentation.

5. Conclusion

In this work, we discussed the problem of individual strength of assumptions in ABA frameworks. We proposed a general framework to rank assumptions based on their strength inside an ABA framework without additional information like a preference order. Additionally, we proposed principles in order to compare different ranking-based semantics based on their behaviour alone. We also defined a family of ranking-based semantics for ABA based on approaches and ideas for AFs. For an ABA framework we construct the corresponding AF then apply known ranking-based semantics in order to rank arguments in the corresponding AF to finally re-interpret this ranking in the ABA setting. We have shown that if the underlying ranking-based semantics for AFs satisfies certain principles, then this family of semantics satisfies our proposed principles for the ABA setting as well.

As for future work, we want to look at other structured argumentation frameworks like ASPIC$^+$ and apply similar ideas in order to rank individual elements of the ASPIC$^+$ framework based on their strength alone. Our current approach uses AFs in order to rank assumptions. As a follow-up we want to propose direct approaches only using the ABA framework without the help of the corresponding AF.

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