Revisiting minimal admissible sets in abstract argumentation

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Abstract. We revisit non-empty minimal admissible sets in abstract argumentation frameworks, also called *initial sets* [7]. These sets are a simple concept for analysing conflicts in an abstract argumentation framework and to explain why certain arguments can be accepted. We contribute with new insights on the structure of initial sets and devise a simple non-deterministic construction principle for any admissible set, based on iterative selection of initial sets of the original framework and its induced reducts.

Keywords: abstract argumentation · admissibility · decomposition

1 Introduction

Formal argumentation [1, 3] encompasses approaches for non-monotonic reasoning that focus on the role of arguments and their interactions. The most well-known approach is that of *abstract argumentation frameworks* [5] that model arguments as vertices in a directed graph, where a directed edge from an argument a to an argument b denotes an *attack* from a to b. Formal semantics are (usually) given to abstract argumentation frameworks by *extensions*, i. e., sets of arguments that can jointly be accepted and represent a coherent standpoint on the conflicts between the arguments.

In this extended abstract, we revisit one of the fundamental concepts underlying approaches to formal argumentation (and abstract argumentation in particular) for the purpose of *explaining*, namely *admissibility*. Informally speaking, a set of arguments is *admissible* if each of its members is defended against any attack from the outside (we will provide formal details in Section 2). Many popular semantics for abstract argumentation rely on the notion of admissibility. In particular, a *preferred extension* is a maximal (wrt. set inclusion) admissible set and preferred semantics satisfies many desirable properties [2]. However, since a preferred extension is a maximal admissible set, it can hardly be used for explaining why a certain argument is acceptable: such an extension may contain many irrelevant arguments and its size alone distracts from the particular reasons why a certain member is acceptable. Our aim is to investigate why certain arguments are contained in, e. g., a preferred extension and how we can decompose such large extensions into smaller sets that allow us to justify the reasoning process behind such complex semantics.

As a tool for our investigation, we consider *initial sets*, i. e., non-empty admissible sets that are minimal wrt. set inclusion. Initial sets have been introduced in [6] and further analysed in [8, 7]. We contribute to this analysis with new insights on the structure of initial sets and, in particular, to the use of initial sets for the task of explanation.

We proceed as follows. Section 2 presents preliminaries on abstract argumentation and Section 3 introduces and further analyses initial sets. We provide a characterisation result of admissible sets in Section 4 that can be used for the task of explanation. Section 5 concludes this extended abstract.

2 Abstract Argumentation

An *abstract argumentation framework* AF is a tuple $\mathsf{AF} = (\mathsf{A}, \mathsf{R})$ where A is a (finite) set of arguments and R is a relation $\mathsf{R} \subseteq \mathsf{A} \times \mathsf{A}$ [5]. For two arguments $a, b \in \mathsf{A}$ the relation $a\mathsf{R}b$ means that argument a attacks argument b. For a set $X \subseteq \mathsf{A}$, we denote by $\mathsf{AF}|_X = (X, \mathsf{R} \cap (X \times X))$ the projection of AF on X. For a set $S \subseteq \mathsf{A}$ we define

$$S^+ = \{a \in \mathsf{A} \mid \exists b \in S, b\mathsf{R}a\} \qquad S^- = \{a \in \mathsf{A} \mid \exists b \in S, a\mathsf{R}b\}$$

We say that a set $S \subseteq \mathsf{A}$ is *conflict-free* if for all $a, b \in S$ it is not the case that $a\mathsf{R}b$. A set S *defends* an argument $b \in \mathsf{A}$ if for all a with $a\mathsf{R}b$ there is $c \in S$ with $c\mathsf{R}a$. A conflict-free set S is called *admissible* if S defends all $a \in S$.

Different semantics can be phrased by imposing constraints on admissible sets. In particular, an admissible set E

- is a *complete* extension iff for all $a \in \mathsf{A}$, if E defends a then $a \in E$,
- is a *grounded* extension iff E is complete and minimal,
- is a *stable* extension iff $E \cup E^+ = \mathsf{A}$,
- is a *preferred* extension iff E is maximal.

All statements on minimality/maximality are meant to be with respect to set inclusion.

3 Initial Sets

Admissibility is a fundamental property for most abstract argumentation semantics and it captures the basic intuition for an explanation *why* a certain argument can be regarded as acceptable. More concretely, if S is an admissible set then $a \in S$ is accepted *because* all arguments S are accepted, every attacker of a is attacked back by some argument in S. However, admissibility alone is not sufficient to model explainability as it does not take *relevance* into account.

Example 1. Consider the argumentation framework AF_0 depicted in Figure 1. There are four admissible sets containing the argument d:

$$S_1 = \{a, d, f, g\} \qquad S_2 = \{d, f, g\} \qquad S_3 = \{a, d, g\} \qquad S_4 = \{d, g\}$$

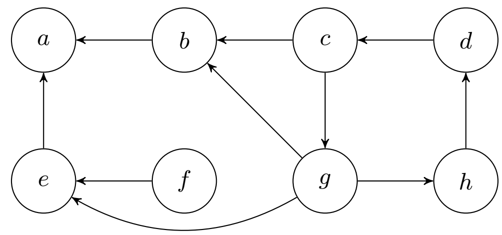

Fig. 1. The argumentation framework AF_0 from Example 1.

S_1 is also a preferred extension. However, it is also clear that arguments a and f are not integral for defending d and the set S_4 presents a concise description of what is needed in order to deem d as acceptable (namely only g and d itself).

In the following, we take relevance into account by considering *minimal* (wrt. set inclusion) admissible sets. Of course, a notion of minimal admissible set without further constraints is not a useful concept as the empty set is always admissible and constitutes the unique minimal admissible set. Non-empty minimal admissible sets have been coined *initial sets* in [6].

Definition 1 (Xu and Cayrol 2016). *For* $\mathsf{AF} = (\mathsf{A}, \mathsf{R})$*, a set* $S \subseteq \mathsf{A}$ *with* $S \neq \emptyset$ *is called an* initial set *if* S *is admissible and there is no admissible* $S' \subsetneq S$ *with* $S' \neq \emptyset$*. Let* $\mathit{IS}(\mathsf{AF})$ *denote the set of initial sets of* AF*.*

Example 2. We continue Example 1. There are three initial sets of AF_0: $\{f\}$, $\{c, h\}$, and $\{g, d\}$.

We now contribute some new results on the structure of initial sets, therefore extending the analysis from [6, 8, 7]. Initial sets have an interesting property with respect to strongly connected components as follows. Recall that we can decompose an abstract argumentation framework AF into its strongly connected components. More precisely, an abstract argumentation framework $\mathsf{AF}' = (\mathsf{A}', \mathsf{R}')$ is a strongly connected component (SCC) of AF, if $\mathsf{AF}' \sqsubseteq \mathsf{AF}$ s.t. there is a directed path between any pair $a, b \in \mathsf{A}'$ in AF' and there is no larger AF'' with that property. Let $\mathsf{SCC}(\mathsf{AF})$ be the set of SCCs of AF. The following result shows that initial sets are always completely contained in a single SCC.

Proposition 1. *If* S *is an initial set of* AF *then there is* $\mathsf{AF}' = (\mathsf{A}', \mathsf{R}') \in \mathit{SCC}(\mathsf{AF})$ *s.t.* $S \subseteq \mathsf{A}'$*.*

The proofs of the above and the following results are omitted due to space limitations.

If S is an initial set let $\mathsf{SCC}(S)$ denote its SCC as in the above proposition. Initial sets can actually be characterised by their SCC as follows.

Proposition 2. S *is an initial set of* AF *if and only if* S *is an initial set of* $\mathit{SCC}(S) = (\mathsf{A}', \mathsf{R}')$ *and* $S^- \subseteq \mathsf{A}'$*.*

In other words, a set S is an initial set iff it is an initial set of a single SCC and it is not attacked by arguments outside the SCC.

4 Characterising Admissibility-based Semantics

In [6, 8] it has been shown that any admissible set (and in particular every complete and preferred extension) can be constructed by 1.) selecting a set of non-conflicting initial sets, 2.) adding further defended arguments, and 3.) iterating this procedure taking so-called "J-acceptable" sets into account. In particular, the described mechanism involves iterative application of the characteristic function [5], computation of the grounded extension, and said notion of J-acceptability to provide those characterisations (and some further concepts). In this section, we provide a (arguably) more elegant formalisation of these ideas.

Our characterisations rely on the notion of the *reduct* [4].

Definition 2. *For* $\mathsf{AF} = (\mathsf{A}, \mathsf{R})$ *and* $S \subseteq \mathsf{A}$*, the* S*-reduct* AF^S *of* AF *is defined via* $\mathsf{AF}^S = \mathsf{AF}|_{\mathsf{A}\setminus(S\cup S^+)}$.

The following results show that by an iterative selection of initial sets on the corresponding reducts, we can re-construct every admissible set (these observations have already been hinted at in [6, 8, 7]).

Theorem 1. *Let* $\mathsf{AF} = (\mathsf{A}, \mathsf{R})$ *be an abstract argumentation framework and* $S \subseteq \mathsf{A}$. S *is admissible if and only if either*

- $S = \emptyset$ *or*
- $S = S_1 \cup S_2$, $S_1 \in \mathit{IS}(\mathsf{AF})$ *and* S_2 *is admissible in* AF^{S_1}.

By recursively applying the above theorem, we obtain the following corollary.

Corollary 1. *Every non-empty admissible set* S *can be written as* $S = S_1 \cup \ldots \cup S_n$ *with pairwise disjoint* S_i, $i = 1, \ldots, n$, S_1 *is an initial set of* AF *and every* S_i, $i = 2, \ldots, n$ *is an initial set of* $\mathsf{AF}^{S_1\cup\ldots\cup S_{i-1}}$.

Let us now discuss the wider significance of Theorem 1 and Corollary 1. For that, recall the standard approach to compute and justify the (uniquely determined) grounded extension of an argumentation framework $\mathsf{AF} = (\mathsf{A}, \mathsf{R})$ [5], cf. also the discussion in [6]. Basically, the grounded extension E of AF can be computed by selecting any non-attacked argument $a \in \mathsf{A}$, add it to E, remove a and all arguments attacked by a from AF (so move from AF to $\mathsf{AF}^{\{a\}}$), and continue the process until no further unattacked argument can be found. Observe the similarity of this procedure to the procedure indicated by Theorem 1: in order to construct any admissible set S of AF, first select any initial set S' of AF, add it to S (which is initially empty), remove S' and all arguments attacked by S' from AF, and continue the process. Therefore, initial sets allow us to *serialise* the construction of any admissible set into smaller steps, each of these steps solving a single conflict in the framework under consideration. Depending on how initial sets are selected at each step and how we end the process, we can also recover different semantics. Let us be more formal and consider the following transition rule on states of the form (AF, S) where AF is an abstract argumentation framework and S is a set of arguments:

$$(\mathsf{AF}, S) \xrightarrow{S' \in \mathsf{IS}(\mathsf{AF})} (\mathsf{AF}^{S'}, S \cup S') \tag{1}$$

If (AF', S') can be reached from (AF, S) via a finite number of steps (this includes no steps at all) with the above rule we write $(\mathsf{AF}, S) \rightsquigarrow (\mathsf{AF}', S')$. We can make the following observations.

Theorem 2. *Let* AF *be an abstract argumentation framework.*

1. *A set* S *is admissible if and only if* $(\mathsf{AF}, \emptyset) \rightsquigarrow (\mathsf{AF}', S)$ *(for some* AF'*).*
2. *A set* S *is complete if and only if* $(\mathsf{AF}, \emptyset) \rightsquigarrow (\mathsf{AF}', S)$ *(for some* AF'*) and* AF' *contains no unattacked argument.*
3. *A set* S *is grounded if and only if* $(\mathsf{AF}, \emptyset) \rightsquigarrow (\mathsf{AF}', S)$ *(for some* AF'*),* AF' *contains no unattacked argument, and a set* S' *can only be selected in (1) if* $S' = \{a\}$ *and* a *is unattacked.*
4. *A set* S *is preferred if and only if* $(\mathsf{AF}, \emptyset) \rightsquigarrow (\mathsf{AF}', S)$ *(for some* AF'*) and* $\mathit{IS}(\mathsf{AF}') = \emptyset$*.*
5. *A set* S *is stable if and only if* $(\mathsf{AF}, \emptyset) \rightsquigarrow ((\emptyset, \emptyset), S)$*.*

Note that item 3 also shows that the standard algorithm to compute the grounded extension is a special case of our transition rule.

Example 3. We continue Example 2. Suppose we first select $C_1 = \{c, h\}$. For the reduct $\mathsf{AF}_1 = \mathsf{AF}_0^{C_1} = (\{a, e, f\}, \{(e, a), (f, e)\}$ we obtain $\mathsf{IS}(\mathsf{AF}_1) = \{\{f\}\}$. If we now select $\{f\}$ we obtain $\mathsf{AF}_2 = \mathsf{AF}_1^{\{f\}} = (\{a\}, \emptyset)$ with $\mathsf{IS}(\mathsf{AF}_2) = \{\{a\}\}$. Selecting now $\{a\}$ we end up with $\mathsf{AF}_3 = \mathsf{AF}_2^{\{a\}} = (\emptyset, \emptyset)$. It is clear that $\{c, h\} \cup \{f\} \cup \{a\} = \{a, c, f, h\}$ is a complete, preferred, and stable extension.

5 Summary and conclusion

We investigated initial sets as a means to decompose the derivation of extensions wrt. certain semantics. This decomposition can, in particular, be used to explain why a certain argument is contained in a specific extension. The initial sets selected in the decomposition show exactly what is needed for a certain argument to be included.

Part of ongoing work consists in a deeper analysis of the concepts of initials sets, as well as deriving characterisations for further semantics based on admissibility. Also the exploitation of these characterisations for algorithmic purposes is part of future work.

Acknowledgements The work reported here has been supported by the Deutsche Forschungsgemeinschaft (project number 375588274).

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